Charles-Augustin Coulomb
First Memoir on Electricity and Magnetism

Charles-Augustin Coulomb, Construction et usage d'une balance électrique, fondée sur la propriété qu'ont les fils de métal, d'avoir une force de réaction de torsion proportionnelle à l'angle de torsion, *Histoire de l'Académie [royale] des sciences avec les mémoires de mathématiques et de physique, partie "Mémoires" [1785]*, 1788, p. 569-577.

Translated by Louis L. Bucciarelli, Emeritus Professor of Engineering and Technology Studies (MIT), MIT, 2000 (revised and notes added by Christine Blondel and Bertrand Wolff, 2012).

Construction and use of an electric Balance based on the property that filaments of metal produce a reactive force in torsion proportional to the angle of twist.

Experimental determination of the law according to which the elements of bodies, electrified with the same kind of Electricity, are mutually repelled.

In a memoir presented to the Academy, in 1784, I have determined from experiments the laws governing the torsional resistance of a filament of metal and I have found that this force is proportional to the angle of torsion, to the fourth power of the diameter of the suspended filament and inversely proportional to its length - all multiplied by a constant coefficient which depends on the nature of the metal and is easily determined by experiment.

I have shown in the same Memoir that by means of this force of torsion, it was possible to precisely measure extremely small forces as, for example, one ten thousandths of a grain[1]. In the same Memoir I described a first application of this theory, seeking to evaluate the constant force attributed to adhesion in the formula for the surface friction of a solid body moving through a fluid.

Today, I set before the eyes of the Academy, an electric balance constructed according to the same principles. It measures with the greatest precision the state and the electric force of a body, however weak the degree of electricity.

Construction of the Balance (Pl. XIII)

While practice has taught me that, in order to execute several electric experiments in a convenient way, it is necessary to correct some defaults in the first

1 1 grain = 0,053 g

balance of this kind that I put to use, still, as this has been up until now the only one which I have employed, I am going to give its description, noting how its form and dimensions can and ought to be changed according to the nature of the experiments one plans to carry out. The first figure presents the balance in perspective. Here are the details.

On a cylinder of glass *ABCD*, of 12 [pouces[2]] in diameter and 12 [pouces] high, we place a plate of glass of 13 [pouces] in diameter, which covers the whole vessel of glass. This cover is pierced with two holes of approximately 20 [lignes[3]] in diameter, one in the middle, at *f*, on which is elevated a tube of glass of 24 [pouces] in height. This tube is cemented over the hole *f*, with the cement used in electric apparel: at the highest extremity of the tube at *h*, is placed a torsion micrometer the details of which are shown in *figure 2*. The top, *n° 1*, bears a knob *b*, the pointer *io*, and the clasp of suspension, *q*. This piece goes into the hole *G* of part *n° 2*. Part *n° 2* is formed of a circle *ab* divided on its edge into 360 degrees, and of a copper tube Φ which fits into the tube *H*, *n° 3*, sealed at the interior at its highest extremity of the tube or of the glass stem *fh* of *figure 1*. The clasp *q* (*figure 2*, *n° 1*) has approximately the shape of the extremity of a solid mechanical pencil clamp [*porte-crayon*], which can be tightened by means of the annulus *q*. The clasp of this *porte-crayon* holds the end of a filament of very thin silver. The other end of the filament of silver is fixed (*fig. 3*) at *P*, by the clasp of a cylinder *Po* of copper or of iron, whose diameter is but a [ligne], and whose end *P* is split and forms a clasp which is tightened by means of the collar Φ. This small cylinder has a hole in C, in order to allow the needle *ag* to slide through (*fig. 1*). It is necessary that the weight of this small cylinder be of sufficient magnitude in order to put the filament of silver in tension without breaking it. The needle that one sees (*fig. 1*) at *ag*, suspended horizontally at approximately the midpoint of the height of the big vase which encloses it, is formed, either of a filament of silk plastered with sealing wax, or of a straw likewise covered with Spanish wax, and finished off from *q* to *a*, a distance of 18 [lignes], by a cylindrical filament of shellac. At the end *a* of this needle, is a small ball of pith of two to three [lignes] diameter. At *g*, is a small vertical plane of paper coated with turpentine which serves as a counter-weight to the ball *a* and which dampens the oscillations.

We have said that the cover *AC* is pierced with a second hole at m. It is in this second hole that one introduces a small cylinder *mΦt*, of which the lower part Φt is shellac. At *t* is a ball likewise made of pith. Around the vase, at the height of the needle, one scribes a circle *zQ* divided into 360 degrees: for simplicity, I have made use of a band of paper divided into 360 degrees, which I glue around the vase, at the height of the needle.

To begin to operate with this instrument, in placing the cover (atop the vase), I position the hole *m* approximately at the first division or at the point O of the circle *zOQ* traced on the vase. I place the pointer *oi* of the micrometer on the point O, at the first division of this micrometer. I then turn the micrometer within the vertical tube *fh*

2 1 pouce = 2,7 cm
3 1 ligne = 0, 23 cm

until, keeping in view the vertical filament which suspends the needle and the center of the ball, the needle *ag* is directed towards the first division of the circle *zOQ*. I then introduce through the hole *m* the other ball *t* suspended by the filament *mΦ t* such that it touches the ball *a* and that, in keeping in view the center of the filament of suspension and the ball *t*, one encounters the first division *o* of the circle *zOQ*. The balance is now in a state ready for all operations; we go on to give as an example, the means by which we are able to determine the fundamental law according to which electrified bodies repel themselves.

The fundamental law of Electricity.

The repulsive force of two small globes electrified with the same kind of electricity, is inversely proportional to the square of the distance between the centers of the two globes.

THE EXPERIMENT

One electrifies, *fig. 4*, a small conductor, which is nothing but a pin with a large head, which is isolated by forcing its point into the end of a rod of Spanish wax. One introduces this pin into the hole *m*, bringing it in contact with the ball *t*, [which is] in contact with the ball *a.* In retracting the pin the two balls find themselves electrified with the same kind of electricity and they repel themselves mutually to a distance that one measures in viewing, (keeping in line) the filament of suspension and the center of the ball a, the division corresponding to the circle *zOQ.* Turning then the pointer of the micrometer in the direction *pno*, one torques the filament of suspension *lp*, and one produces a force proportional to the angle of torsion, which tends to make the ball *a* approach the ball *t*. One observes, by this means, the distance to which different angles of torsion bring the ball *a* back towards the ball *t* and in comparing the forces of torsion with the corresponding distances of the two balls, one determines the law of repulsion.

I will only present here, some tests which are easy to repeat and which will immediately reveal the law of repulsion.

First Test. Having electrified the two balls with the head of the pin, with the pointer of the micrometer positioned at *o*, the ball *a* of the needle is displaced from the ball *t* by 36 degrees.

Second Test. Having torqued the filament of suspension by means of the knob *o* of the micrometer by 126 degrees, the two balls approach each other and stop at 18 degrees distance the one from the other.

Third Test. Having torqued the filament of suspension by 567 degrees, the two balls approach until 8 and one-half degrees.

Explication and results of this experiment.

Before the balls are electrified, yet touching, the center of the ball *a*, fixed to the needle, is at a distance equal to the diameter of the balls from the point where the torsion of the filament of suspension is null. It is necessary to be warned that the filament of silver *lP*, 28 [pouces] long, which forms the suspension is so fine that the weight of one pied[4] of length is but 1/16 of a grain. In calculating the force required to twist this filament, in acting at the point *a* elongated some four [pouces] from the filament *lP* or from the center of suspension, I have found, using the formulas derived in a Memoire on the laws of the force of torsion of filaments of metal, printed in the volume of the Academie for 1784, that to torque this filament 360 degrees, requires at point *a*, in acting with the lever *aP* of four [pouces] of length, a force of 1/340 of a grain. Thus as the forces of torsion are, as proved in the Memoire, proportional to the angles of torsion, the least repulsive force between the two balls, displaces them sensibly one from the other.

We find in our first test, where the pointer of the micrometer is at the point *o*, that the balls are displaced 36 degrees, which produces in the same time a force of torsion of
36° = 1/3400 of a grain;
in the second test, the distance of the balls is 18 degrees, but as one has torqued the micrometer 126 degrees, it results that at the distance of 18 degrees, the repulsive force is 144 degrees: thus at half of the first distance, the repulsion of the balls is quadrupled.

In the third test, where we have twisted the filament of suspension 567 degrees, the two balls find themselves no further apart than 8 and one half degrees. The total torsion, is consequently, 576 degrees, quadruple the one of the second test, and it is only off by one half a degree that the distance of the two balls in this third test was reduced to half of the distance it was in the second. It results thus from these three tests, that the repulsive action of the two balls electrified with the same kind of electricity exert on upon the other was the inverse ratio of the square of the distances.

First Remark

In repeating the preceding tests, we will observe that in making use of a filament of silver, as thin as the one we have employed (which only requires a torsional force of approximately 24 thousands of a grain to twist it through an angle of 5 degrees) that however calm be the air, and whatever precautions that one can take, we could not answer for the natural position of the needle when the torsion is zero, to within 2 or 3 degrees. Thus, in order to have a first test to compare with the following ones, it requires, after having electrified the two balls, to torque the filament of suspension some 30 to 40 degrees, this which will give a force of torsion strong enough so that the

4 1 pied = 32,5 cm

2 or 3 degrees of uncertainty in the initial position of the needle, when the torsion is zero, does not produce any sensible error in the results. It is necessary furthermore to be warned that the filament of silver, which I used in this test, is so fine that it breaks with the least disturbance. I have found in the following that it would be more useful to employ in these tests a filament of suspension of nearly double the diameter, although its flexibility to torsion be 14 to 15 times smaller than that of the first. It is necessary to take care, before making use of this filament of silver, over the course of two or three days, to tension it by a weight which is approximately half this that might break it. It is necessary still yet to warn the reader, that in using this last filament of silver, never to torque it beyond 300 degrees because in exceeding this degree of torsion it begins to strain-harden and reacts, as we have proven in the Memoir already cited, printed in 1784, with a force smaller than the one corresponding to the torsion angle.

Second Remark

The electricity of the two balls diminishes somewhat over the duration of the experiment. I noticed that, the day where I have made the preceding tests, the electrified balls, finding themselves repulsed to 30 degrees one from the other, under an angle of torsion of 50 degrees, they come back toward each other about one degree in three minutes. But as I have only used two minutes to make the three preceding tests, one can, in these tests, neglect the error which results from the loss of electricity. If one desires greater precision, as when the air is humid, and the electricity dissipates rapidly, one ought, by a preliminary observation, determine the law of diminution of the electric action of the two balls in each minute, and then, on the basis of this preliminary observation, use it to correct the results of tests that one wishes to make that day.

Third Remark

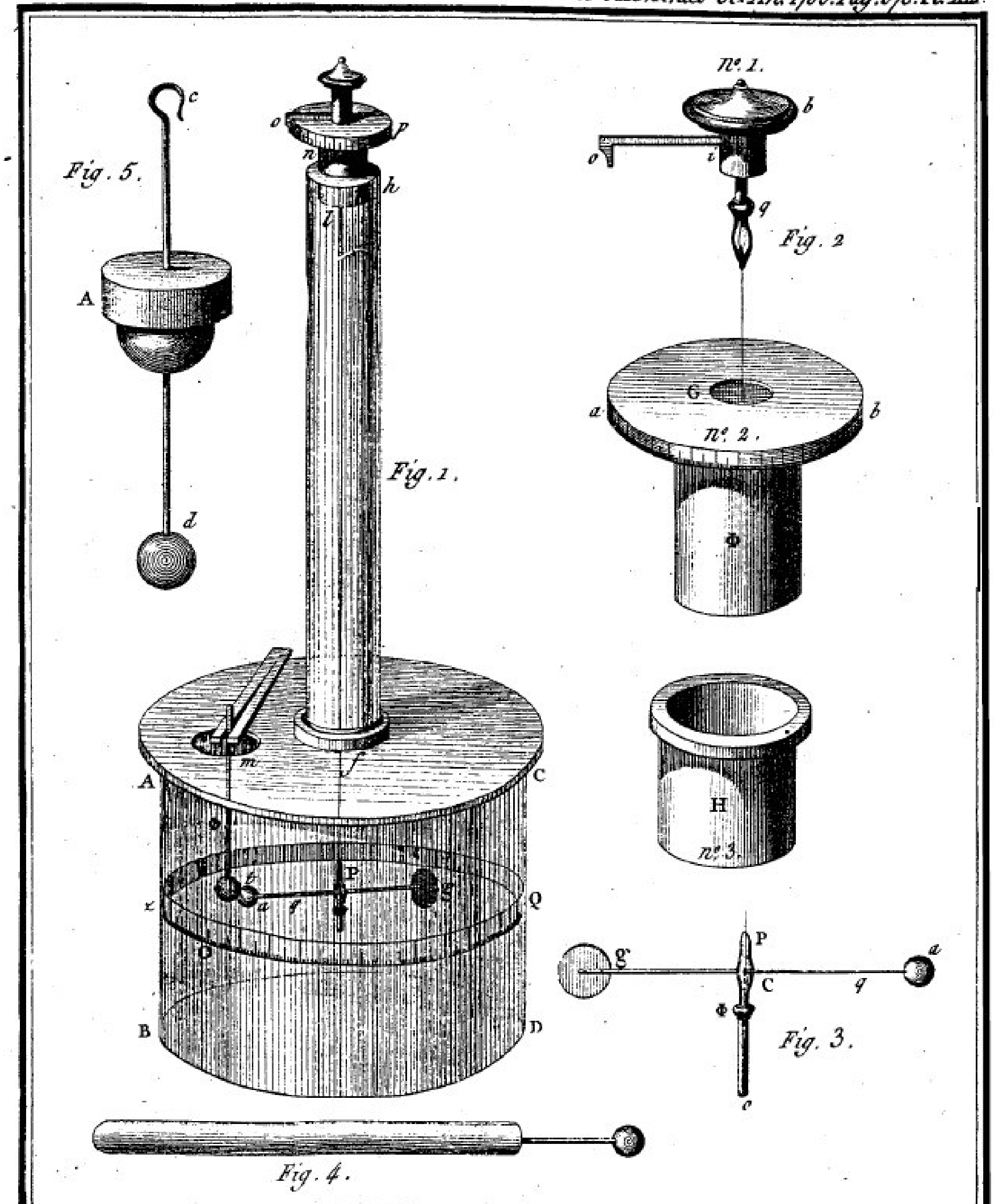
The distance of the two balls, when they are displaced one from the other by their reciprocal repulsive action, is not precisely measured by the angle they make, but by the cord of the arc which joins their centers. In the same way that the lever at whose extremities acts the action, is not measured by the mean of the length of the needle, or by the radius, but by the sine of the half of the angle formed by the distance of the two balls. These two quantities, of which one is smaller than the arc and diminishes consequently the distance measured by this arc, while the other reduces the lever arm, compensating themselves in some way; and in the tests of this sort with which we are concerned, one can without sensible error, hold to the evaluation that we have given, if the distance of the two balls does not exceed 25 to 30 degrees; in the other cases, one must make the rigorous calculation.

Fourth Remark

As experience has shown, in a well closed chamber, one can determine with the first filament of silver, to within 2 or 3 degrees, the position of the needle, when the torsion is null, this which gives, after the calculation of the forces of torsion, proportional to the angle of torsion, a force more or less of a 40 thousandth of a grain, the weakest degree of electricity will be measurable easily with this balance. For this operation, one makes pass, *fig 5*, across a cork of Spanish wax, a small filament of copper *cd*, terminating at *c* by a crochet and in *d*, by a small ball of gilded pith, and we put the cork *A* in the trough *m* of the balance, *fig 1*, in such a way that the center of the ball *d*, viewed by the string of suspension, retakes to point *o* of the circle *zOQ*. In approaching then an electrified body of the crochet *o*, however weak be the electricity of this body, the ball *a* separates from the ball *d*, giving the sign of the electricity and the distance of the two balls as a measure of the force, according to the principle of the inverse ratio of the square of the distances.

But I ought to warn you that, since these first tests, I have had different small electrometers made according to the same principles of the force of torsion, using a filament of silk for the suspension, such as it leaves the cocoon, or a thread of the sheep of Angora. One of these electrometers which has almost the same shape as the electric balance, described in this Memoir, is much smaller. It is only 5 to 6 [pouces] in diameter, a stem of one pouce; the needle is a small filament of shellac of 12 [lignes] of length, terminated at *a* by a small very light circle of tinsel.

. The needle and the tinsel weigh a little more than a quarter of a grain; the filament of suspension, such as it leaves from the cocoon, is 4 [pouces] long, having a flexibility such that in acting with a lever arm of one pouce, it only requires a sixtieth thousands of a grain to twist it an entire circle or 360 degrees. In presenting in this electrometer at the crochet *C* of *figure 5*, an ordinary rod of Spanish wax, electrified by friction, at a 3 feet distance from this crochet, the needle is chased to more than 90 degrees. We will describe in more detail in the following this electrometer, when we will determine the nature and the degree of electricity of different bodies which throuhg rubbing each other, take on a very weak degree of electricity.



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