XX.

SECOND SECTION

On the alteration of the elastic force in the torsion of filaments of metal. Theory of the coherence and of elasticity.

When one torques the filaments of iron or of brass, stretched, as in the preceding experiments, by a weight, one observes two things; if the angle of torsion is not so great, relative to the length of the filament of suspension, at the moment when one releases the weight, it returns approximately to the position that it had before twisting, that is to say, the filament of suspension untwists completely by the quantity by which it had been torqued; but if the angle of torsion given the suspending filament, is very large, then the filament only unwinds a certain amount, & the center of the reaction of torsion will advance the whole quantity by which the filament failed to unwind. It follows from these two considerations, that two suites of experiments are required; the first to determine, by the diminution of oscillations, how much the elastic force of torsion is altered in oscillatory movement under conditions in which the center of reaction of torsion is not displaced; the second to determine the displacement of this center of reaction, when the angle of torsion is sufficiently large for this displacement to take place. [256]

XXI.

FIRST EXPERIMENT

Filament of iron, n.^{o} 1, length, six pouces six lignes

We have taken a filament of iron of six *pouces* six *lignes* in length, that has been loaded with a weight of two *livres*, the same as has served us in the experiments in the preceding section. In turning the cylinder about its axis in order to twist the filament of suspension, we have sought to determine how many degrees the amplitude diminishes with each oscillation, & we have found:

First test, angle of torsion	90^{d}	loss of 10^{d} in	3 1/2 oscillat.
Second test	45	....	10 1/2
Third test	22 1/2	....	23
Fourth test	11 1/4	....	46

Remarks on this Experiment

The reductions in amplitudes of oscillations have been very uncertain (irregular), when the initial angle of torsion was more than 90 degrees; we have even observed that in this case, in twisting the cylinder about its axis, it did not return to its initial position, and the respective position of the constitutive parts of the filament have been altered, and and consequently, its center of reaction of torsion has remained displaced: here is what the experiments gave for this displacement.

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Follow on to the first Experiment.

In this part of the first experiment, we have searched to determine the displacement of the center of torsion, due to the degree of torsion that we have given to the filament of suspension. [257]

First test, in twisting the filament	1/2 C. {the index or the center of torsion has been displaced .....}	8^d
Second test	1 C	50
Third test	2 C	310
Fourth test	3 C	1C + 300
Fifth test	4 C	2C + 290
Sixth test	5 C	3C + 280
Seventh test	6 C	4C + 260
Eighth test	10 C	8C + 240

Ninth test. Having wished to continue to twist the filament some 15 new circles, always in the same sense, it broke at the fourteenth. After this experiment, this filament was straight and very rigid, it had separated along its length into two parts; examining it with a magnifying glass, this separation was very evident and it had exactly the shape of a cord formed of two (torons) helices.

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Remarks concerning this Experiment.

This first experiment and its sequel appears to show that below 45 degrees, the alterations made are approximately proportional to the amplitudes of the angles of twist, as one sees from the second, third and fourth tests of the first experiment; that above 45^d, the alterations augment in a ratio much greater; that the center of reaction twist only begins to displace when the angle of torsion is approximately a half circumference; that this displacement increases as the torsion of the filament increases; that it is very irregular up to 1 circle 10 degrees; and that, passing this level of torsion, the reaction of torsion remains [258] approximately the same for all the angles of twist: Thus, for example, in the fourth test, in twisting the filament three circles, the center of reaction of torsion displaces one circle + 300 degrees, so the reaction of torsion has only led the cylinder back one circle 60 degrees. In the seventh test, we see that after having already experience in the previous tests a [total] displacement of more than eight circles, that six new circles of torsion displace the center of reaction of torsion by 4C + 260 degrees, so that for more than fourteen circles of torsion, the reaction of torsion is still only one circle plus 100 degrees; thus it only differs by a tenth from the reaction of torsion for the fourth test which gave us one circle + 60 degrees: the experiments which follow clarify this remark.

XXIV

SECOND EXPERIMENT

Filament of iron, n.^{o} 7, length, 6 pouces 6 lignes.

We have searched, in the first part of this experiment, how much the amplitudes of oscillations diminish at each oscillation, when the center of torsion is not yet displaced.

First test, angle of torsion	180^d	loss of 10^d in	3 1/2 oscillat.
Second test	90		12
Third test	45		27
Fourth test	22 1/2		54

Follow on to this second Experiment.

In this second part of the same experiment, we have sought the diplacement of the center of torsion. [259]

First test, in twisting the filament	3 C.	{the index or the center of torsion has been displaced .....}	300^d
Second test	4 C		1C + 180
Third test	6 C		3 + 90
Fourth test	8 C		5 + 90
Fifth test	12 C		9 + 40
Sixth test	20 C		16 + 310
Seventh test	30 C		26 + 180
Eighth test	50 C		46 + 20
Ninth test	at seventeenth circle of torsion, the filament broke.		

XXV.

THIRD EXPERIMENT

Filament of iron, n.^{o} 12, length, 6 pouces 6 lignes.

The first part of this experiment has been made in accord with the first part of the two preceeding experiments.

First test, angle of torsion	360^d	loss of 10^d in	1 oscillat.
Second test	180	10	2
Third test	90	10	5
Fourth test	45	10	11
Fifth test	22 2/2	10	25

Follow on to this third Experiment.

Displacement of the center of torsion.

First test, in twisting the filament	4 C.	{the index or the center of torsion has been displaced .....}	300^d
Second test	6 C		2C + 40

[260]

Third test after six other turns the filament broke.

XXVI

The preceding experiments have been continued with the filaments of brass used in the experiments of the first section.

FOURTH EXPERIMENT

Filament of brass, n.^{o} 1, length, 6 pouces 6 lignes.

First test, in twisting	189^d	loss of 12^d in	2 oscillat.
Second test	90	10	6
Third test	45	10	16
Fourth test	22 2/2	10	40
Fifth test	11 3/4	10	80

Follow on to the fourth Experiment

Displacement of the center of torsion.

First test, in twisting the filament	2 C. {the index or the center of torsion has been displaced ....}	160^d
Second test	4 C	2C + 0
Third test	6 C	3 + 300
Fourth test	10 C	7 + 300
Fifth test	20 C	17 + 340
Sixth test	at the twenty-eighth circle of torsion the filament broke.	

FIFTH EXPERIMENT

Filament of brass, n.^{o} 7, length, 6 pouces 6 lignes.

First test, in twisting	360^d	loss of 10^d in	2 1/2oscillat.
Second test	180	10	6
Third test	90	10	13
Fourth test	45	10	31
Fifth test	22 1/2	10	72

Follow on to the fifth Experiment [261]

Displacement of the center of torsion.

In twisting the filament four circles, the center is displaced 220 degrees; but in wishing to torque it six circles, the filament broke.

XXVII

In the filament employed in this last experiment, the torsion altered the oscillations, and hence the elastic force, less than in all the other experiments; it is this which occasions the great number of oscillations before the oscillatory movement dies out; it is this likewise which results in the sudden rupture of this filament, without being able to displace its center of reaction one circle. I have found in general that the filaments of brass, those available in commerce, between the n.^{os} 5 & 8, were those whose elasticity in torsion was the least imperfect: in comparing the filaments of iron & of brass with the *same numbers*, we have similarly found that the filaments of brass have an amplitude of elasticity much more extensive than the filaments of iron.

For the rest, the experiment presents many irregularities in the results: two bobbins of the same filament & and of the same number, do not always give the same displacement for the same angle of torsion, this which can only be attributed to the way in which the filaments are manufactured - to the more or less great pressure that they experience in passing under the (levre de la filiere), to the heat treatment given them in order to successively reduce the diameter from one number to the next, from large to small.

XXVIII

First Remark

Despite the uncertainty which reigns in the experiments of oscillations for the amplitudes of the (etendues), it appears that below certain limits, these alterations are approximately proportional to the amplitude of oscillation, as we have announced in the remarks on the first experiment, & as all the other experiments confirm. The resistance of the air can only alter the amplitude of oscillations very little in our experiments. I am assured of this by the following. The weight of two *livres*, which has served us in the experiments of this section, was 26 lignes in height & 19 lignes in diameter. I have formed with a very light paper, a cylindrical surface of the same diameter as this weight, but which with 70 lignes of height: I put a part of the cylinder of lead into my envelope of paper, & formed thus a cylinder of 78 lignes of height, or three times longer than the first, which should have tripled, in the oscillatory movement, the alterations due to the air resistance; but I have never found that these alterations were a tenth more considerable in the second case as in the first; most often they are equal; thus the resistance of air enters into our experiments only as quantities that one can neglect. [262]

XXIX

Second Remark

In order to make a torsion balance, it is always necessary to choose the filaments which have the least imperfect elasticity; the filaments of brass are much more preferable to those of iron: the choice of the thickness depends on the forces which one wishes to measure. I have a magnetic balance which will be described in our Memoirs, where I alternatively made use of a filament of brass of 3 pieds in length, no.s 12 & 7; the elastic force of torsion is such that in twisting the filaments eight circles, over the course of thirty hours, there is not one degree of alteration or displacement in the center of torsion.

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[263]

Third Remark

In all the filaments of metal, the behavior is elastic only up to a certain point: The isochrony of the oscillations teaches us that in the first degrees of torsion, the elastic force is almost perfect; but beyond the angle of torsion which serves, for thus to say, as a measure of the elastic force, the center of reaction of torsion displaces nearly the whole of all the angle of torsion which exceeds this of the elastic reaction. However, as one can note in the preceding experiments, the amplitude of the elastic reaction is not a constant quantity for all angles of twist, it increases as the torsion increases; the less the initial elasticity in the filament subjected to test, the greatest is this increase. A filament of brass, no. 1, of 6 and one-half pouces in length, made red in a fire, in order to make it loose, by heat treatment the greatest part of its elasticity, only gives, after this operation, for the first circle of torsion, 50 degrees of reaction of elasticity; but it has acquired, after 90 circles of torsion, an elastic extension of nearly 500 degrees in this interval; From the 2nd to 3rd circle of torsion, the reaction of elasticity increases 12 degrees; from the 40 to 41st circle of torsion, the same reaction increases 6 degrees; and from 90 to 91st circle of torsion, almost a degree, such that the increase of the elastic reaction, after the center of reaction has been displaced a certain angle, is nearly inversely proportional to the angle of displacement. It is necessary to point out that after these 90 circles of torsion, I wished to twist the same filament another 50 circles, but it broke at the 49th, so this filament, before breaking, could [264] be twisted to 140 circles. If we compare this result with that which followed from the first experiment, where the same filament, no. 1, had not been heat treated, we found that after 25 circles of torsion, the reaction of elasticity was 480 degrees and that in twisting 15 new circles, the filament fractured; this last filament can thus only take, without breaking, 40 circles of torsion. In following in this experiment the path of the elastic reaction, we deduce from it that at the point of rupture, this reaction is almost equal to that of the heat treated filament in the same point of rupture; from which it would appear that one is justified in concluding that by torsion alone one can give to a heat treated filament all the elasticity it may get and that strain hardening can’t add more to it; such that reciprocally, if in passing our wire through the die or by any other means we have been able to give to our filament of brass a cold working such that its elastic reaction had been 520 degrees, which appears to me to be this of our two filaments at the moment of rupture, for then the elastic reaction had been carried to its maximum by this first operation: There would not have been any more possible displacement in the center of the reaction of torsion; but all the time that we would have made to test this filament to a torsion of more than 520 degrees, it would break.

X XX I

Fourth Remark

From the preceding experiments, this, it appears, is how we can explain the elasticity and coherence of metals. The integral parts of the filament of iron or of brass, or of any metal, have an elasticity that one can regard as perfect, that is to say, that the forces necessary to compress or dilate these integral parts are proportional to the dilatations or compressions they experience; but they are only tied together by the coherence, a constant quantity and absolutely different from the elasticity. In the first stages of torsion, the integral parts change their shape (figure), elongating or compressing, without the points by where they adhere together changing position because the force required to produces these first stages of torsion is considerably less than the force of adhesion; but when the angle of torsion becomes such, that the force with which these parts are compressed or dilated is equal to the coherence which unites these integral parts, then they ought to separate or slide one on the other. This sliding of parts takes place in all ductile bodies but if by this sliding of parts, the ones on the others, the bodies compress, the extent of the points of contact increases and the extent of the domain of elasticity becomes greater. However as these integral parts have a determined figure, the extent of the points of contact can only increase up to a certain degree, beyond which the body breaks; it is this which explains the detailed facts of the preceding article. This which proves again that it is necessary to distinguish the cause of elasticity from the adhesion is that one can vary the coherence at will by the degree of heat treatment without altering in any way the elasticity. It is thus the case when I heated to white my no. 1 filament of copper in the preceding experiments, it lost a great part of its force of coherence: before heat treatment, it could carry up to the point of rupture 22 livres [1 livre = 489.5g] and after the heat treatment it only carried 12 to 14 (livres); but while the adhesion was diminished nearly by half by the heat treatment and the amplitude of elasticity was nearly diminished in the same proportion, however in all the extent (etendue) of the elastic reaction which remained in the heat treated filament, the elasticity was the same, at equal angle of torsion, as in the same filament not heat treated, since in suspending to one and the others the same weights, the time of the same number of oscillations was exactly equal in the two cases. [265]

XXXII

An equally interesting effect due to the deformation of parts coming closer when filaments of metal are twisted is this which takes place when one twists a filament of iron, which by this operation alone acquires as parts come closer, the quality of taking the magnetism to a higher degree than it had before. Here is the experiment which revealed this to me; I have taken a filament of iron, such as one finds them throughout the world of commerce, of the thickness of those which serve for the small sounding bars (sonnettes); a length of six pouces, weighing 57 grains; this filament of six pouces, magnetized (aimante) and suspended horizontally by a filament of silk, untwisted (detordu) and very fine, makes an oscillation in 18 seconds: this same filament of six pouces in length, twisted up to the point of rupture and magnetized as in the first case to saturation by the method of double touching, makes an oscillation in 6 seconds; such that the moment of the directive force for the two needles equal and similar, being as the interval of the square of the times for the same number of oscillations, the magnetic moment of the twisted needle, being nine times more considerable than that of the needle not twisted: I will have the occasion to return to this article in another Memoir.

XXXIII

To confirm all the preceding theory regarding the coherence and elasticity, I have made the following experiments.

We have fixed, *fig. 4,* by means of a C clamp *CD* with a vise *V,* a lamina of steel *AB* on the edged of a very solid table; this beam being pressed and held in its part Aa, between two plates of iron *E* and *F* by the vise *V*: this lamina was 11 lignes long and one-half ligne thick form the point a to the point *B* where was suspended the weight *P*, there was seven pouces of length: we measured on the vertical rule rg, how much the weight *P* made lower [267] the lamina *AB* at its extremity *B*. Here are the details of the results which took place following the different weights with which the lamina was loaded.

We had had the lamina warmed to white hot and then quenched by a very quick cooling; then we have attached at B at seven pouces from point a, different weights. The extremity B has deflected,

With a weight of a half-livre some	8 lignes
With a weight of one livre some	15 1/2
With a weight of one livre and a half some	23 +

We have taken this same lamina and we have heated it until it took on a violet color and it returned to the consistency of an excellent sprint; and we have found equally, that in loading it as in the first case, the extremity B has deflected,

With 1/2 livre de	8 lignes
With 1 livre	15 1/2 +
With 1 1/2 livre	23 +

Finally we have made red this same lamina to white and let it cool (refroidir) very slowly; and we have had, in loading the extremity B, exactly te same results as in the two preceding experiments.

It appears to us that these three experiments prove in an incontestable manner, that whatever the state of the lamina, the first degrees of its elastic force are in no way altered; since in taking account of the lever arm, which diminishes as the lamina is loaded as the lamina is loaded, the same weights deflect it in the three states equally and proportionally to the load; and that when one removes the weights, it retakes exactly it original horizontal position.

I have wished to see subsequently what be the force of this lamina in these three different states; and in the case where the center of flexure would begin to displace, what would [268] be the degree of flexure where the lamina would begin to be deformed without returning to its original position. Here is the result of this experiment.

I have cut from a sheet of English steel, three lamina exactly similar to this of the preceding experiment: one of these lamina have been quenched, the second had been returned to the consistency of an excellent spring, and the third had been heat treated to white and slowly cooled. I attached, fig. 4, a weight d at 2 pouces and 1/2 distant from the point a and I had carefully exerted a pull always perpendicular to the direction of the lamina. Here is what I observed.

The lamina which had been rapidly quenched broke under a pull of six livres; but under whatever angle what it was deflected below this of rupture it returned exactly to its original position. The lamina returned to a violet color, forming an excellent spring, broke only under a pull of eighteen livres; it bent up to the point of rupture, with an angle nearly proportional to the angle of torsion, and under any angle that it was bent before rupture, when we freed it, it retook its original position. The lamina heat treated to white and slowly cooled, bent up to a pull of five to six livres, proportionally to this force of pull, and with an angle absolutely equal under the same force that in the state of quenched and of spring; but in pulling always subsequently perpendicular to the direction of the lamina, in order to conserve the same lever, with a force of seven livres, we have bent it under all the angles, without that it was necessary to augment this force: in letting go, it raised itself back up only by the quantity of which it had been originally deflected by a pull of six livres; such that the angle of reaction of flexure, found itself changed from all the angle which we had bent it with a force greater than seven livres. [269]

These last experiment lead us back to the same results as those which went before. It is clear that in order to have an idea of what happens in the flexure of metals, it is necessary to distinguish the elastic force of the integral parts from the force of adhesion which ties these parts together: The elastic force depends, as we have already said, on the compression or dilation that the integral parts experience and is always proportional to the tractions. These integral parts are not altered, neither by the quenching nor by the heating, since we see that in theses different states, the elasticity is the same under the same degrees of flexure; but these integral parts, are only tied among themselves by a certain degree of adhesion which probably depends on their shape and on the respective portion of the different fluids with which their pores are filled, this which varies according to the quenching and the heating. In the quenched (trempe roide) steel and in the good springs the integral molecules can neither slide one on the other nor experience the least displacement without the body breaking; but in the ductile bodies, in the heat treated metals, these parts can slide one on the other and displace themselves, without the adhesion being sensibly altered.

This that we have come to explain for metals appears to be able to be applied to all bodies; their parts are always of a perfect elasticity, but the bodies made hard, mous or fluids, follow the adhesion of the integral parts. If in the hard bodies, they can slide one upon the other, without their distance being sensibly altered, the body will be ductile or malleable; but if they can not slide one on the other, without their respective distances being sensibly altered, the bodies break when the force with which the bodies will be pulled or compressed, will be equal to the adhesion.

[*fin*]

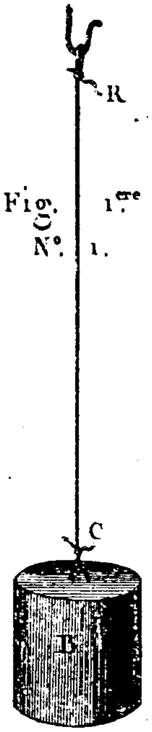
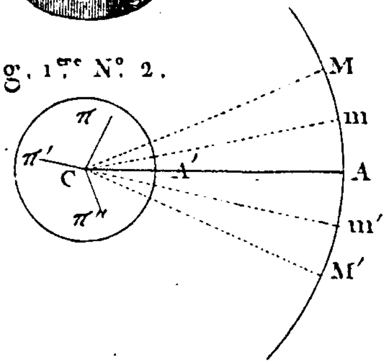


Fig. 1re N°. 1.

Fig. 1re N°. 2.



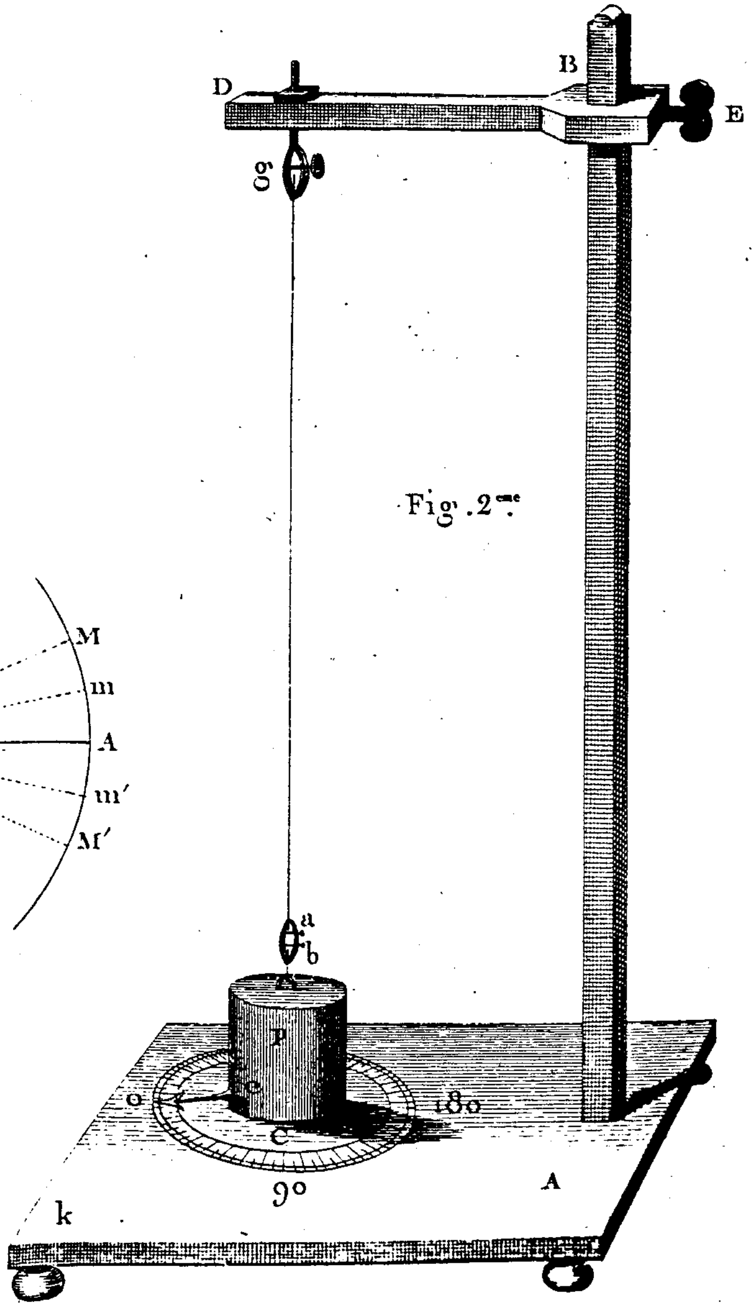


Fig. 2me.

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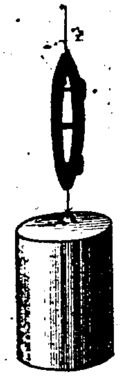
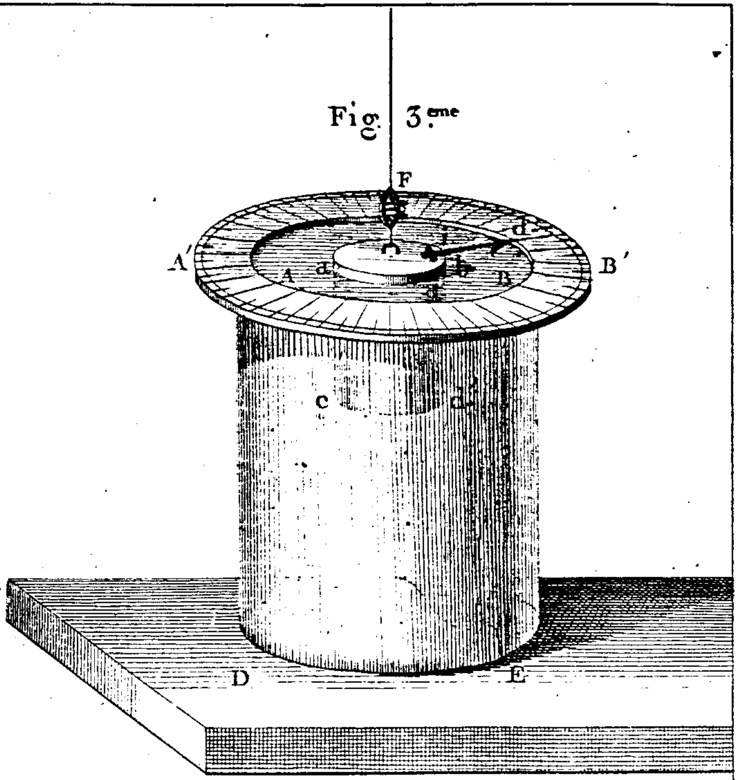


Fig. 3.eme



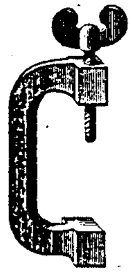
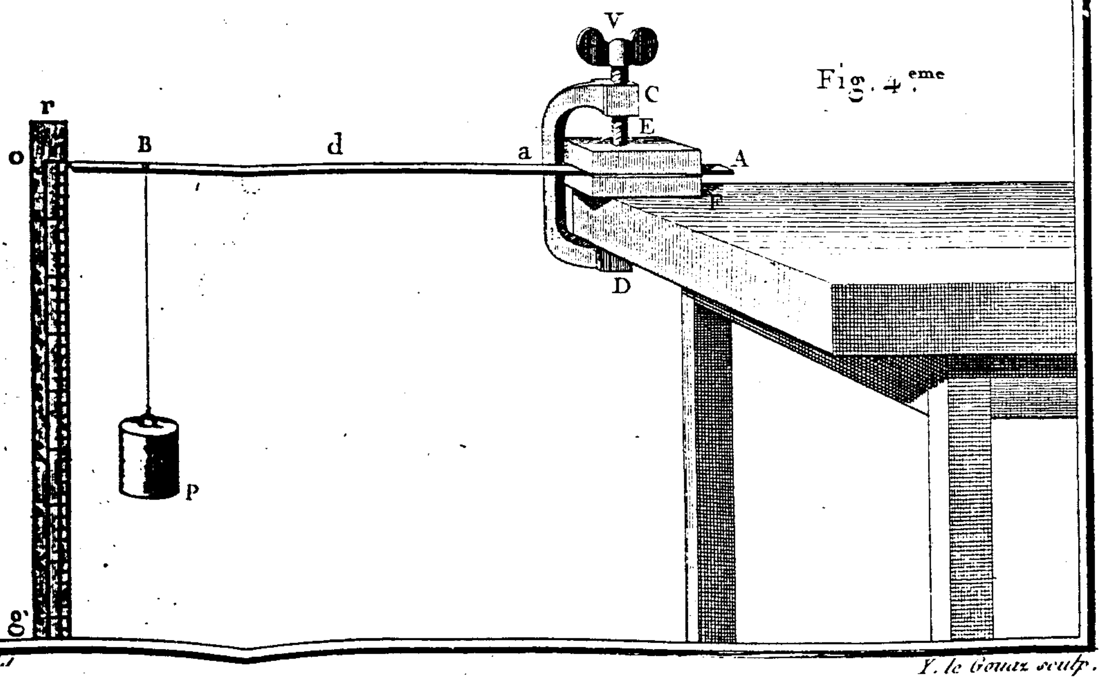


Fig. 4.eme



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