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THE VARIOUS BRANCHES OF SCIENCE,

THE LIBERAL AND FINE ARTS,

AGRICULTURE, MANUFACTURES,

AND COMMERCE.

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"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Monit. Polit.* lib. i. cap. 1.

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one direction only, and the movement of continuous rotation is a necessary consequence of it. It is useless to add, that if the actions exerted by the horizontal conductor upon the two portions of the moveable conductor (which I have just mentioned) force it to turn in opposite directions, it is because the current from this last conductor cannot approach that of the horizontal conductor in one of those portions without diverging from it in the other, and *vice versâ*. However, as the manner in which I have established the relation

$$2k + n = 1$$

was not, perhaps, sufficiently rigorous, as I had verified it only on a describing current, either an entire or a semi-circumference, whereas it ought to have been done upon each *element* of the circular horizontal current, I have therefore produced another instrument, by which the same relation between n and k may be obtained in a more simple manner, and the inconvenience which I have just spoken of is avoided; because the experiment which I make with that instrument proves at once that the action of a complete circuit on an element of the electric fluid is always perpendicular to the direction of this element, which is sufficient to demonstrate that $2k + n = 1$, as I shall show in a note which I intend to publish shortly, and where the description of the instrument here presented will be found.

Paris, Aug. 16th, 1825.

LX. *Memoir on a new Electro-dynamic Experiment, on its Application to the Formula representing the mutual Action of the two Elements of Voltaic Conductors, and on some new Results deduced from that Formula.* By M. Ampere.*

THE manner in which I have determined the relation between the two co-efficients of the formula by which I represented the mutual action of the two elements of electric currents, in the memoir which I read before the Academy on the 10th of June 1822, being liable to some difficulties, I have endeavoured to establish this relation in a more simple and direct manner. I succeeded in this very easily by means of an instrument which I shall first describe; I will then present some new results which I have deduced from this formula.

On a stand TT′ Plate I. (fig. 3) in the shape of a table, two columns

* From the *Annales de Chimie et de Physique*, tom. xxix. p. 381. This memoir was read at the Royal Academy of Sciences of Paris, at the sitting of the 12th of September last.

columns EF, E'F', are raised, connected with one another by means of two transverse bars LL', FF'; an axis G H is maintained between these two bars in a vertical position. Its two extremities G, H, terminating in sharp points, enter into two conical apertures, situated one in the lower transverse bar LL', the other in the end of a screw KZ, borne by the upper transverse bar FF', and destined to press on the axis GH without forcing it. In C is firmly fixed to this axis an arm QCO, of which the extremity O presents a hinge, in the centre of which is engaged an arc of a circle AA' formed by a metal wire which always remains in a horizontal position, and the radius of which is the distance from the point O to the axis. This axis is balanced by a counterpoise Q, for the purpose of lessening the friction of the axis GH in the conical apertures in which its extremities are received.

Above the arc AA' are arranged two small troughs full of mercury, in such a manner that the surface of the mercury rising above the edges shall always come within the arc AA' in B and B'. These two troughs communicate by means of metallic conductors MN M'N' with cups P, P' full of mercury. The cup P and the conductor MN which unites it with the trough M, are fixed to a vertical axis entering into the table so as to be capable of turning freely. The cup P', to which is fixed the conductor M'N', is intersected by the same axis, round which it turns as independently as the other. It is isolated from it by means of a glass tube V which covers this axis, and by a glass shield U which separates it from the conductor of the little trough M, in a manner that the conductors MN, M'N' may be placed at any angle that may be desired.

Two other conductors IR, I'R' fixed to the table are respectively immersed in the cups P, P', and make them communicate with cavities R, R' made in the table and filled with mercury. A third cavity S, also filled with mercury, is between the two others.

The following is the process for using this apparatus: Immerge one of the rheophors (for instance, the positive) into the cavity R, and the negative into the cavity S, which is put in communication with the cavity R' by a curved conductor of any shape. The current will follow the conductor RI, pass into the cup P, thence into the conductor NM, into the trough M, into the portion BB' of the arc AA', in the trough M', the conductor M'N', the cup P', the conductor I'R', and, at last, from the cavity R' into the curved conductor which goes into the cavity S in which the negative rheophor is plunged.

According

According to this disposition the whole of the voltaic circuit is composed:

1. Of the arc BB′ and the conductors MN, M′N′.

2. Of a circuit formed by the parts RIP, P′I′R′ of the apparatus, by the curved conductor going from R′ to S, and the pile itself.

This last circuit must act as a complete one, since it is not interrupted except by the thickness of the glass which isolates the two cups P, P′: it will therefore be sufficient to observe its action on the arc BB′ in order to establish by experiment the action of a complete circuit upon an arc in the different positions we may give it.

When, by means of the joint O, the arc AA′ is placed in such a position that its centre is outside the axis G H, this arc begins to move, and slides on the mercury of the little troughs MM′ by the force of the action of the complete curved current, which runs from R′ into S. If on the contrary its centre is in the axis, it remains immoveable: the complete circuit has therefore no action to make it turn round the axis, and that whatever be the size of the part BB′ determined by the opening of the angle of the conductors MN, M′N′. If, therefore, we take two arcs BB′ differing little from each other, as the momentum of rotation is null for either of them, it will be null for their little difference, and therefore for every element of the circumference, the centre of which is in the axis; whence it follows that the direction of action which the complete circuit exercises on the element, passes through this axis, and is thus perpendicular to the element.

When the arc AA′ is situated so that its centre is in the axis, the portions of the conductors MN, M′N′ exercise on the arc BB′ equal and opposite repulsive actions, in such a manner that no effect can result from it; and since there is no motion, we are sure that there is no momentum of rotation produced by the complete circuit.

When the arc AA′ moves in the other situation in which we supposed it first, the actions of the conductors MN and M′N′ are no longer equal. One might be led to believe that the motion is owing only to this difference; but in proportion as we approach or remove the curved conductor running from R′ to S, the movement is increased or diminished; which circumstance leaves no room for doubt that the complete circuit bears a great share in the effect noticed.

If we once establish by this experiment that the action of a complete circuit on an element of the voltaic circuit is always perpendicular to the direction of this element, we may, by a very simple calculation, deduce from it the relation between

tween n and k, which I had before found by another process. It is sufficient for this purpose to decompose the action which each of the elements of the complete circuit exercises on the element in consideration, into two forces; the one perpendicular to this element, and the other which shall have the same direction with itself, and which I shall call the *elementary tangential force:* then to sum up all the elementary tangential forces produced by the complete circuit, and to equal this sum with zero, which is the tangential force due to the whole circuit. Thus then, if we represent by $d s'$ the element on which it acts, by $d s$ an element of this same circuit, and otherwise preserve the denominations of the memoir printed in the *Annales de Chimie et de Physique*, tome xx., p. 398, et seq. we shall have for the mutual action of the two elements,

$$- i\, i'\, r^{1-n-k}\, d\,(r^k\, d' r \quad \text{(page 413)};$$

moreover

$$\cos\beta = - \frac{d\, r}{d\, s'} \text{ (page 408)},$$

whence

$$d' r = \frac{d\, r}{d\, s'} d\, s' = - d s' \cos\beta,$$

which changes the expression of this action into

$$i\, i'\, d s'\, r^{1-n-k}\, d\,(r^k \cos\beta);$$

for $d\, s'$ which represents the element on which the complete circuit acts, is constant with respect to the characteristic d.

In order to have the elementary tangential force, we must multiply this value by $\cos\beta$, which gives

$$i\, i'\, d\, s'\, r^{1-n-k} \cos\beta\, d\,(r^k \cos\beta),$$

which may be put under the form

$$\tfrac{1}{2} i\, i'\, d s'\, r^{1-n-2k}\, d\,(r^k \cos\beta)^2.$$

Integrating by parts, we obtain for the total of the tangential force

$$\tfrac{1}{2} i\, i' d s' \left\{ r^{1-n-2k} (r^k \cos\beta)^2 - (1-n-2k) \int r^{-n-2k} (r^k \cos\beta)^2 d r \right\};$$

or

$$\tfrac{1}{2} i\, i'\, d\, s' \left\{ \frac{\cos^2\beta}{r^{n-1}} - (1-n-2k) \int \frac{\cos^2\beta}{r^n}\, d\, r \right\}.$$

As the circuit is closed, r and β will take the same value at the limits; thus the first part

$$\frac{\cos^2\beta}{r^{n-1}}$$

will disappear. But it will not be the same with the second, which cannot be calculated till we have replaced one of the variables r and β by its value in the function of the other drawn from

from the equations of the circuit, so that we may choose these equations in such a manner that the integral

$$\int \frac{\cos^2 \beta}{r^n} \mathrm{d}\, r$$

is not reduced to zero between the limits. In order to remove the total of the tangential force, it is necessary that the coefficient of this integral be null; which gives the relation sought for, $2\,k + n - 1 = 0$.

In order to form a juster idea of the integral

$$\int \frac{\cos^2 \beta}{r^n} \mathrm{d}\, r,$$

we may conceive, round the middle of the element d s taken for a centre, an infinity of spherical surfaces, which divide the complete circuit into infinitely small arcs, so that the two extreme spherical surfaces touch it at the two points of this circuit, which are, one the furthest from, and the other the nearest to, the middle of the element; then we may consider the complete circuit as being composed of two branches terminating at these two points, and both divided into an equal number of infinitely small arcs, so that every arc of one branch corresponds with that of the other branch comprised between the two same consecutive spherical surfaces: for two corresponding arcs we have then the same value of r, and the values of d r are equal, but of contrary signs, for the current cannot go, in withdrawing from the element d s' into one of the branches, without going, in approaching it, into the other. Thence we see why the integral $\int f(r)\, \mathrm{d}\, r$ is always null when it is taken in the whole extent of the complete circuit, since this integral is then composed of elements which are, two by two, of equal value, but of different signs.

It would be the same with $\int f(r) \cos^2 \beta\, \mathrm{d}\, r$, if $\cos^2 \beta$ had the same value for any two corresponding elements; *ex. gr.* if these two elements were always situated symmetrically on the two sides of a plane raised perpendicularly on the middle of d s'; but if, on the contrary, in one of the two branches the absolute value of $\cos \beta$ for every element is greater than for its correspondent, $\int f(r) \cos \beta\, \mathrm{d}\, r$ will be composed of two series of terms, one of which will contain only positive terms, and the other negative terms; so that each of the former shall have an absolute value greater or smaller than that of the negative term corresponding with it in the other series. Then this integral can never be null; and in order to make the tangential force comformable to experience, we must have $2\,k + n - 1 = 0$.

Setting out from this relation between k and n, and naming β' and β'', r' and r'', these values of β and r which correspond with the two extremities of a portion of the voltaic conductor, we find, for the action which it exercises on the element ds' in the direction of this element

$$\tfrac{1}{2} i i' ds' \left[\frac{\cos^2 \beta''}{r''^{n-1}} - \frac{\cos^2 \beta'}{r'^{n-1}}\right],$$

or rather

$$\tfrac{1}{2} i i' ds' \left[\frac{\cos^2 \beta''}{r''} - \frac{\cos^2 \beta'}{r'}\right],$$

since we know from other experiments that $n = 2$. It is sufficient to change the sign of this expression, which is independent of the form of the portion of the voltaic conductor, and only depends on the situation of its two extremities with respect to the element ds', in order to have the force with which the same portion of the conductor is drawn in a contrary direction by the element following a right line parallel to the direction of the latter; whence it follows that if this element forms a part of a fixed rectilinear conductor, we shall have the value of the force which the whole conductor exercises, in order to move that portion of which we are speaking, in a direction parallel to this conductor, by integrating between the limits marked by its two extremities the value which we have just found for the tangential force of the element ds'.

If we call a' and a'' the lowered perpendiculars of the two extremities of the portion of the conductor which we consider as moveable, on the rectilinear conductor which we have to calculate the action parallel to its direction, we shall have

$$r'' = \frac{a''}{\sin \beta''}, \quad r' = \frac{a'}{\sin \beta'},$$

$$ds' = -\frac{d' r''}{\cos \beta''} = \frac{a'' d\beta''}{\sin^2 \beta''} = -\frac{d' r'}{\cos \beta'} = \frac{a' d\beta'}{\sin^2 \beta'},$$

and consequently,

$$\frac{ds'}{r''} = \frac{d\beta''}{\sin \beta''}, \quad \frac{ds'}{r'} = \frac{d\beta'}{\sin \beta'};$$

whence it is easy to conclude that the integral sought for is

$$-\tfrac{1}{2} i i' \int \left[\frac{\cos^2 \beta'' \, d\beta''}{\sin \beta''} - \frac{\cos^2 \beta' \, d\beta'}{\sin \beta'}\right]$$

$$= -\tfrac{1}{2} i i' \left[l \, \frac{\operatorname{tang} \frac{1}{2} \beta''}{\operatorname{tang} \frac{1}{2} \beta'} + \cos \beta'' - \cos \beta' + C \right].$$

We must take this integral between the limits determined by the two extremities of the rectilinear conductor; by calling $\beta_{\prime}'$, $\beta_{\prime}''$, $\beta_{\prime\prime}'$, $\beta_{\prime\prime}''$ the values of β' and of β'' relative to those limits, we have immediately that of the force exercised by the rectilinear conductor, and that last value evidently depends only on the four angles $\beta_{\prime}'$, $\beta_{\prime}''$, $\beta_{\prime\prime}'$, $\beta_{\prime\prime}''$.

When

When we want the value of this force in a case where the rectilinear conductor extends indefinitely in the two directions, we must make $\beta_{/}' = \beta_{/}'' = 0$, and $\beta_{//}' = \beta_{//}'' = \pi$: it seems at first sight that then it becomes null, which would be contrary to experience; but we easily see that the part of the integral in which are the cosines of these four angles, is the only one which vanishes in this case, and that the rest of the integral

$$\frac{1}{2}\, i\, i' \left[1\, \frac{\tang \frac{1}{2}\beta_{/}''}{\tang \frac{1}{2}\beta_{/}'} - 1\, \frac{\tang \frac{1}{2}\beta_{//}''}{\tang \frac{1}{2}\beta_{/}''} \right]$$

$$= \frac{1}{2}\, i\, i'\, 1\, \frac{\tang \frac{1}{2}\beta_{/}'' \cot \frac{1}{2}\beta_{//}''}{\tang \frac{1}{2}\beta_{/}' \cot \frac{1}{2}\beta_{/}''},$$

becomes $$\frac{1}{2}\, i\, i'\, 1\, \frac{\tang^2 \frac{1}{2}\beta_{/}''}{\tang^2 \frac{1}{2}\beta_{/}'} = i\, i'\, 1\, \frac{\tang \frac{1}{2}\beta_{/}''}{\tang \frac{1}{2}\beta_{/}'} = i\, i'\, 1\, \frac{a''}{a'}.$$

This value shows that the force sought for then only depends on the relation of the two perpendiculars a' and a' lowered on the rectilinear and indefinite conductor of the two extremities of that portion of the conductor on which it acts; that it is also independent of the form of this portion, and only becomes null, as it ought, when the two perpendiculars are equal to themselves.

In order to have the distance of this force from the rectilinear conductor, the direction of which is parallel to its own, we must multiply every one of the elementary forces of which it is composed by its distance from the conductor, and integrate the result with reference to the same limits; we shall thus have the momentum to be divided by the force in order to obtain the distance sought for.

We easily find, after the above values, that the value of the elementary momentum is

$$\frac{1}{2}\, i\, i'\, \mathrm{d}\, s'\, r \sin\beta\, \mathrm{d}\left(\frac{\cos^2\beta}{r}\right).$$

This value cannot be integrated but by substituting for one of the variables r or β its value in the function of the other, drawn from the equations which determine the form of the moveable portion of the conductor. It becomes very simple when this portion is found on a right line elevated on some point of the rectilinear conductor, which is considered as perpendicularly fixed in its direction, because in taking this point as the origin of s', we have

$$r = -\frac{s}{\cos\beta},$$

because s' is a constant relatively to the differential

$$\mathrm{d}\, \frac{\cos^2\beta}{r};$$

the value of the elementary momentum therefore becomes

$$\frac{1}{2}\, i\, i'\, \mathrm{d}\, s'\, \frac{\sin\beta}{\cos\beta}\, \mathrm{d}\,(\cos^3\beta) = -\frac{3}{2}\, i\, i'\, \mathrm{d}\, s' \sin^2\beta \cos\beta\, \mathrm{d}\,\beta,$$

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the integral of which, between the limits β'' and β' is

$$-\tfrac{1}{2}\, i i'\, ds' \left[\sin^2 \beta'' - \sin^2 \beta'\right].$$

Replacing ds' by the values of this differential found above, and integrating again, we have between the limits determined by the two extremities of the rectilinear conductor,

$$\tfrac{1}{2}\, i i' \left[a'' (\cos \beta_{\prime\prime}'' - \cos \beta_{\prime}'') - a' (\cos \beta_{\prime\prime}' - \cos \beta_{\prime}')\right].$$

If we suppose that this conductor extends indefinitely in the two directions, we must give to $\beta_{\prime}'$, $\beta_{\prime}''$, $\beta_{\prime\prime}'$, $\beta_{\prime\prime}''$ the values which we have already assigned them in this case, and we shall have

$$-\, i i' \, (a'' - a')$$

for the value of the momentum of rotation, which will consequently be proportionate to the length $a'' - a'$ of the moveable conductor, and will not change, so long as that length remains the same, whatever may be the distances of the extremities of this latter conductor from that which is considered fixed.

It is easy to see that this value is that of the momentum of rotation which the fixed conductor imparts to the portion of another rectilinear conductor, situated on a right line which intersects the direction of the former at right angles, for the purpose of making it turn round the point of intersection of the directions of the two conductors. If we descend from the top of the right angle thus formed, to their intersection, by the direction of the two currents from the perpendiculars upon the four right lines which join, two by two, the extremities of these currents, and if we represent these perpendiculars by $p_{\prime}'$, $p_{\prime}''$, $p_{\prime\prime}'$, $p_{\prime\prime}''$, we shall have

$$p_{\prime}' = \pm\, a' \cos \beta_{\prime}',\; p_{\prime}'' = \pm\, a'' \cos \beta_{\prime}'',\; p_{\prime\prime}' = \pm\, a' \cos \beta_{\prime}'',$$
$$p_{\prime\prime}'' = \pm\, a'' \cos \beta_{\prime\prime}'',$$

according as the current from the conductor which has been considered as fixed, is approaching or withdrawing from the point where the direction of this conductor meets that of the other; and the value of the momentum of rotation, with which it tends to revolve round this point of the moveable conductor, becomes consequently

$$\pm\, \tfrac{1}{2}\, i i' \,(p_{\prime\prime}'' - p_{\prime}'' - p_{\prime\prime}' + p_{\prime}'),$$

that is to say, precisely the same as if it were produced by four forces equal to $\frac{1}{2}\, i i'$; of which two would be attractive and in the direction of the right lines which join the extremities of the same name of the two conductors, and the two others repulsive and acting in the direction of the right lines which join the extremities of different names of the same conductors.

If the currents extend to the point of intersection of the directions of the two conductors, three of these four perpendiculars will be null, and the momentum of rotation will be simply proportional to the height of the right-angled triangle of which

which these two conductors will be the sides; so that if it be supposed that their length be increased or diminished in the same proportion, the momentum of rotation will also be increased or diminished in the same proportion.

The result we have just obtained is but a particular case of the general value of the momentum of rotation resulting from the mutual action of two rectilinear conductors $L'L''$, $L_{,}L_{,,}$ (fig. 4), situated in the same plane, in order to make each other revolve round the point of intersection O of their directions. In order to calculate more easily the value of this momentum, which we shall call M, we shall place that of the mutual action of the two elements d s, d s' under this form,

$$\tfrac{1}{2}\, i\, i' \frac{d\, s'}{\cos\beta}\, d\left(\frac{\cos^2\beta}{r}\right),$$

which results immediately from the circumstance that the component of this action, in the direction of the element d s', becomes

$$\tfrac{1}{2}\, i\, i'\, d\, s'\, d\left(\frac{\cos^2\beta}{r}\right),$$

as we have just seen, in making $k = -\frac{1}{2}$ and $n = 2$.

If we take the point of intersection of the directions of the two conductors for the origin of the distances $OM = s$, $OM' = s'$, we shall have $s' \sin\beta$ for the perpendicular OP lowered from this point on the right line which joins the centres of the two elements, and for the value of the elementary momentum of rotation,

$$\frac{d^2 M}{d\, s\, d\, s'}\, d\, s\, d\, s' = \tfrac{1}{2}\, i\, i'\, s'\, d\, s' \operatorname{tang}\beta\, d\left(\frac{\cos^2\beta}{r}\right),$$

whence it is concluded,

$$\frac{d\, M}{d\, s'}\, d\, s' = \tfrac{1}{2}\, i\, i'\, s'\, d\, s' \left(\frac{\sin\beta\cos\beta}{r} - \int \frac{d\,\beta}{r}\right).$$

But according to the manner in which the angles have been taken in the formula representing the mutual action of the two elements of voltaic conductors, the angle β is external to the triangle OMM'; and by calling ε the angle MOM' comprised between the directions of the two currents, the third angle OMM', equal to α, will also be so to $\beta - \varepsilon$, which gives

$$r = \frac{s' \sin\varepsilon}{\sin(\beta - \varepsilon)},$$

we have therefore

$$\frac{d\, M}{d\, s'}\, d\, s' = \tfrac{1}{2}\, i\, i' \frac{d\, s'}{\sin\varepsilon} [\cos\beta \sin\beta \sin(\beta - \varepsilon) + \cos(\beta - \varepsilon) + C].$$

Replacing in this value $\cos(\beta - \varepsilon)$ by

$$\cos^2\beta \cos(\beta - \varepsilon) + \sin^2\beta \cos(\beta - \varepsilon);$$

it will easily be seen that it is reduced to

$$\frac{d\, M}{d\, s'}\, d\, s' = \tfrac{1}{2}\, i\, i' \frac{d\, s}{\sin\varepsilon} [\cos\varepsilon \cos\beta + \sin^2\beta \cos(\beta - \varepsilon) + C],$$

which

which we must take between the limits β' and β''. We have also the difference of the two functions of the same form, one of β'', the other of β', which must be again integrated, in order to obtain the rotation sought for: it is enough to make this second integration upon one of these two quantities: let a'' then be the distance OL″ which answers to β'', we shall have

$$s' = \frac{a'' \sin(\beta'' - \varepsilon)}{\sin \beta''} = a'' \cos \varepsilon - a'' \sin \varepsilon \cot \beta'';\ d s' = \frac{a'' \sin \varepsilon\, d\beta''}{\sin^2 \beta''};$$

and the quantity which we shall wish to integrate first will be

$$\tfrac{1}{2} a'' i i' \left\{ \frac{\cos \varepsilon \cos \beta''\, d\beta''}{\sin^2 \beta''} + \cos(\beta'' - \varepsilon)\, d\beta \right\},$$

the integral of which taken between the limits $\beta''_{,}$ and $\beta''_{,,}$ is

$$\tfrac{1}{2} a'' i i' \left\{ \sin(\beta_{,,}'' - \varepsilon) - \sin(\beta_{,}'' - \varepsilon) - \frac{\cos \varepsilon}{\sin \beta_{,,}''} + \frac{\cos \varepsilon}{\sin \beta_{,}''} \right\}.$$

Designating by $p_{,,}''$ and $p_{,,}'$, the perpendiculars lowered from the point O on the distances $L''L_{,,} = r_{,,}''$, $L''L_{,} = r_{,}''$, we have evidently

$$a'' \sin(\beta_{,,}'' - \varepsilon) = p_{,,}'',\ a'' \sin \beta_{,}'' = p_{,}'',\ \frac{a''}{\sin \beta_{,,}''} = \frac{r_{,,}''}{\sin \varepsilon},\ \frac{a''}{\sin \beta_{,}''} = \frac{r_{,}''}{\sin \varepsilon},$$

and the preceding integral becomes

$$\tfrac{1}{2} i i' \left[p_{,,}'' - p_{,}'' - (r_{,,}'' - r_{,}'') \cot \varepsilon \right].$$

If we notice that by designating the distance OL′ by a', we have also

$$s' = \frac{a' \sin(\beta' - \varepsilon)}{\sin \beta'} = a' \cos \varepsilon - a' \sin \varepsilon \cot \beta',\ d s' = \frac{a' \sin \varepsilon}{\sin^2 \beta'},$$

we easily see that the integral of the other quantity is formed by that which we have just obtained, changing $p_{,,}''$, $p_{,}''$, $r_{,,}''$, $r_{,}''$ into $p_{,,}'$, $p_{,}'$, $r_{,,}'$, $r_{,}'$, which gives for the value of the momentum of rotation which is the difference of the two integrals

$$\tfrac{1}{2} i i' \left[p_{,,}'' - p_{,,}' - p_{,,}' + p_{,}' - (r_{,,}'' - r_{,,}' - r_{,,}' + r_{,}') \cot \varepsilon \right].$$

That value is reduced to what we have found above, in the case where the angle ε is right, because then $\cot \varepsilon = 0$.

If we suppose that two currents proceed from point O, and that their lengths OL″, $OL_{,,}$ (fig. 5) are respectively represented by a and b, the perpendicular O P by p, and the distance $L''L_{,,}$ by r, we shall have

$$\tfrac{1}{2} i i' \left[p + (a + b - r) \cot \varepsilon \right],$$

for the value which, in this case, the momentum of rotation takes.

The quantity $a + b - r$, the excess of the sum of two sides of a triangle on the third, is always positive; whence it follows that the momentum of rotation is greater than the value $\tfrac{1}{2} i i' p$ which it takes when the angle ε of the two conductors is a right

right one, whilst cot ε is positive, *i. e.* whilst this angle is acute; but it becomes smaller when the same angle is obtuse, because then cot ε is negative. Moreover, it is evident that its value becomes so much greater as the angle ε is smaller, and that it increases *ad infinitum* like cot ε in proportion as cot ε approaches zero; but it will be well to show that it always remains positive, however near this angle be to two right lines.

For that, it is sufficient to observe that by calling α the angle of the triangle $OL''L_{\prime\prime}$ comprised between the sides a and r, and β that which is between the sides b and r, we have $\cot \varepsilon = -\cot(\alpha + \beta)$, $p = a \sin \alpha = b \sin \beta$, $r = a \cos \alpha + b \cos \beta$, and consequently

$$a + b - r = a(1 - \cos \alpha) + b(1 - \cos \beta)$$
$$= p \operatorname{tang} \tfrac{1}{2} \alpha + p \operatorname{tang} \tfrac{1}{2} \beta,$$

and

$$\tfrac{1}{2} i i' [p + (a + b - r) \cot \varepsilon] = \tfrac{1}{2} i i' p \left(1 - \frac{\operatorname{tang} \frac{1}{2} \alpha + \operatorname{tang} \frac{1}{2} \beta}{\operatorname{tang} (\alpha + \beta)}\right),$$

a value which always remains positive, however small the angles α and β, since tang $(\alpha + \beta)$, for inferior angles at $\frac{\pi}{4}$, is always larger than tang α + tang β, and of course more so than tang $\frac{1}{2}\alpha$ + tang $\frac{1}{2}\beta$. This value evidently tends towards the limit $\frac{1}{2} i i' p$ in proportion as the angles α and β approach zero: it vanishes with p when these angles become null.

Departing from this expression at the momentum of rotation resulting from the mutual action of the two rectilinear conductors situated in the same plane, round the point of intersection of their direction, and of the general fact proved again by the experiment described in the beginning of this Memoir, of the nullity of action of a conductor bent in an arc upon a portion of the circuit, the two extremities of which are in the perpendicular raised in the centre of this arc upon the plane on which it is described, I have devised an instrument founded on the same principle as that which I presented about two years ago to the Academy of Sciences, and described in my *Recueil d'Observations Electro-dynamiques*, p. 224, &c. Its object is also to determine by the number of oscillations of a moveable conductor, the value of the action which a fixed conductor exercises upon it; but which has not the inconvenience that was found in the first, of giving the experimental measure of this action in a case in which it cannot be determined, by the aid of my formula, but by calculations of the most complicated kind. I intend soon to publish the description of this instrument.

When the point of intersection of the conductors OL'', $L_{\prime}L_{\prime\prime}$

$L_{\prime} L_{\prime\prime}$ (fig. 6) is found at one of the extremities of the first and in the middle of the second, we obtain the momentum of rotation resulting from the mutual action of these two conductors, with the addition of those referring to each of the angles $L_{\prime} O L''$, $L_{\prime\prime} O L''$, of which the two cotangents are equal and of a contrary sign; so that in marking the distances $L_{\prime\prime} L''$ and $L_{\prime} L''$ by r and r', and the perpendiculars OP, OP' by p and p', we have for that momentum

$$\tfrac{1}{2} i i' [p + p' + (r' - r) \cot \varepsilon].$$

Let us moreover suppose that the length $OL'' = a$ of the conductor which has one of its extremities in O is equal to half $OL_{\prime}$ or $OL_{\prime\prime}$ of the other, and let us call θ the half POL'' or $POL_{\prime\prime}$ of the angle $L_{\prime\prime} O L'' = \varepsilon$, we shall find

$$p = a \cos \theta, p' = a \sin \theta, r = 2 a \sin \theta, r' = 2 a \cos \theta,$$

$$\cot \varepsilon = \frac{1 - \text{tang}^2 \theta}{2 \,\text{tang}\, \theta} = - \frac{1 - \cot^2 \theta}{2 \cot \theta}:$$

the value of the momentum of rotation therefore is

$$\tfrac{1}{2} a i i' \left\{ \cos \theta - \sin \theta \frac{1 - \text{tang}^2 \theta}{\text{tang}\, \theta} + \sin \theta - \cos \theta \frac{1 - \cot^2 \theta}{\cot \theta} \right\},$$

or

$$\tfrac{1}{2} a i i' [\cos \theta \,\text{tang}^2 \theta + \sin \theta \cot^2 \theta] = \tfrac{1}{2} a i i' [\sin \theta \,\text{tang}\, \theta + \cos \theta \cot \theta].$$

It is sufficient to double the expression, suppressing the denominator 2, for that produced by the action of the two conductors $L' L''$, $L_{\prime} L_{\prime\prime}$ of the same length, and the centres of which are at the point O round which one of them is supposed to be moveable.

In the instrument of which I have just spoken, there are two rectilinear conductors of equal length, moveable round their centres; from each of these centres, and sufficiently apart that there may not be between the conductors a sensible mutual action, project two other rectilinear conductors half the length of the others; these are fixed, and form between themselves an angle that may be varied at will: the same electric current runs through the six conduetors; so that in every one of the fixed ones, and in that part of the corresponding moveable conductor nearest to it, its current is in a contrary direction, in order that the latter may keep in a steady equilibrium in the perpendicular direction on the right line, which divides into two equal parts the angle of the two fixed conductors whose action it experiences. As it is this latter angle which is given immediately above the graduated arc attached to one of these fixed conductors, it is desirable to introduce its half, which we represent by η, instead of θ in the expression of the momentum of rotation

$$M = \tfrac{1}{2} a i i' (\sin \theta \,\text{tang}\, \theta + \cos \theta \cot \theta)$$

which

which every fixed conductor impresses on the moveable conductor on which it acts, if we observe that

$$\sin^3\theta + \cos^3\theta = (\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta),$$

and that $\theta = \frac{1}{2}\varepsilon = \frac{1}{2}\left(\frac{\pi}{2} - \eta\right)$

gives

$\sin\theta\cos\theta = \frac{1}{2}\sin\varepsilon = \frac{1}{2}\cos\eta$ and $\sin\theta + \cos\theta = \sqrt{2}\cos\frac{1}{2}\eta$,

we shall have $M = \frac{1}{\sqrt{2}} a\, i\, i' \cos\frac{1}{2}\eta\left(\frac{2}{\cos\eta} - 1\right).$

When the moveable conductor is displaced ever so little from the situation of equilibrium, the angle θ becomes $\theta + d\theta$ with reference to one of the fixed conductors, and $\theta - d\theta$ with reference to the other; so that the difference of the two momenta, which was null in that situation, becomes, after being displaced,

$$2\frac{dM}{d\theta}d\theta = -a\,i\,i'(\cos\theta - \sin\theta)\left(\frac{1}{\sin^2\theta\cos^2\theta} + \frac{1}{\sin\theta\cos\theta} + 1\right)d\theta.$$

This value is always negative when we take, as we suppose it here, the angle ε double of θ on that side where this angle is acute, and consequently after the opposite direction of the electric currents in the two sides of this angle, which requires that the momentum M should tend to increase it, the momentum $2\frac{dM}{d\theta}d\theta$ will tend to diminish it on the side where $d\theta$ is positive, and to increase it on the side where the same differential is negative; *i. e.* to bring back the moveable conductor to the position of equilibrium, which might besides easily be seen *à priori*.

If we introduce in the value just obtained the angle η instead of θ, we find

$$2\frac{dM}{d\theta}d\theta = -a\,i\,i'\sqrt{2}\sin\frac{1}{2}\eta\left(\frac{4}{\cos^2\eta} + \frac{2}{\cos\eta} + 1\right)d\theta.$$

This momentum must be measured, either by the torsion of a thread, or by the number of oscillations made by the two moveable conductors, in a given time, by means of observations made simultaneously * upon these conductors, when we wish to verify the results deduced from my formula, comparing them with those of experience.

I have also devised another apparatus which may serve for the same verifications, by calculating, and afterwards measuring

* By comparing among themselves measures determined by successive operations, we also completely avoid the inaccuracy produced by the variations of the energy of the pile, which necessarily alter the results deduced from experiment. We may also measure in a direct manner the momentum M by the torsion of a thread.

suring, the angles which a rectilinear conductor, moveable round its centre, forms, in the situation of equilibrium, with two fixed conductors, the directions of which are through this centre, and one of which is traversed several times by the electric current, which traverses the other but once. It is thus we obtain the relation which ought to subsist between the momentum of rotation; and we verify it easily if the value of these momenta, calculated after my formula, agrees with experience.

If the conductor, the length of which has been designated by b, were to extend *ad infinitum*, the other, whose length is always $2\,a$ and its middle situated on the direction of the first, we should have

$$p = p' = a \sin \varepsilon, r' - r = 2\,a \cos \varepsilon,$$

and the value of the momentum of rotation

$$\tfrac{1}{2}\, a\, i\, i' \left[p + p' + (r' - r) \cot \varepsilon\right]$$

would become $a\, i\, i' \left[\sin \varepsilon + \frac{\cos^2 \varepsilon}{\sin \varepsilon}\right] = \frac{a\, i\, i'}{\sin \varepsilon}.$

When the fixed conductor is indefinite in its two directions we must double this value, and we have

$$\frac{2\,a\,i\,i'}{\sin \varepsilon}$$

for the momentum of rotation which it gives to the moveable conductor $2\,a$. This momentum is therefore reciprocally proportional to the sine of the angle ε formed by the directions of the two conductors.

The expression $\tfrac{1}{2}\, i\, i'\, \mathrm{d}\, s' \left(\frac{\sin \beta \cos \beta}{r} - \int \frac{\mathrm{d}\, \beta}{r}\right)$,

which I gave (in 1822) in my *Recueil*, page 331, for the value of the component perpendicular to the element $\mathrm{d}\, s'$, may serve to calculate very easily the mutual action of two parallel conductors; for, by calling a the distance of these two conductors, we have first $r = \frac{a}{\sin. \beta}$,

which gives, for the value of the preceding integral between the limits β', β'',

$$\tfrac{1}{2}\, i\, i' \frac{\mathrm{d}\, s'}{a} \left(\cos \beta'' \sin^2 \beta'' + \cos \beta'' - \cos \beta' \sin^2 \beta' - \cos \beta'\right),$$

then at each limit, presenting the values of s by b' and b'',

$$s' = b'' - a \cot \beta'' = b' - a \cot \beta', \mathrm{d}\, s' = \frac{a\, \mathrm{d}\, \beta''}{\sin^2 \beta''} = \frac{a\, \mathrm{d}\, \beta'}{\sin^2 \beta'}.$$

By substituting these values and integrating again between the

the limits $\beta_{,}'$, $\beta_{,,}'$ and $\beta_{,}''$, $\beta_{,,}''$, we have, for the value of the force sought for,

$$\tfrac{1}{2} i i' \Big(\sin\beta_{,,}'' - \sin\beta_{,}'' - \sin\beta_{,,}' + \sin\beta_{,}' - \frac{1}{\sin\beta_{,,}''} + \frac{1}{\sin\beta_{,}''} + \frac{1}{\sin\beta_{,,}'} - \frac{1}{\sin\beta_{,}'}\Big),$$

or

$$\tfrac{1}{2} i i' \Big(\frac{a}{r_{,,}''} - \frac{a}{r_{,}''} - \frac{a}{r_{,,}'} + \frac{a}{r_{,}'} + \frac{r_{,}'' + r_{,,}' - r_{,,}'' - r_{,}'}{a}\Big).$$

If the two conductors are of the same length, and perpendicular to the right lines which join by pairs their extremities on the same side, we have

$$r_{,}' = r_{,,}'' = a, \text{ and } r_{,,}' = r_{,}'' = c.$$

Calling c the diagonal line of the rectangle formed by these two right lines and the direction of the two currents, the foregoing expression then becomes

$$i i' \Big(\frac{c}{a} - \frac{a}{c}\Big) = \frac{i i' l^2}{a c}.$$

Calling l the length of the conductors, and this rectangle becoming a square, we have $\frac{i i'}{\sqrt{2}}$ for the value of the force finally, if we suppose one of the conductors indefinite in the two directions, and that l be the length of the other, the terms in which $r_{,}'$, $r_{,,}'$, $r_{,}''$, $r_{,,}''$ are in the denominator, will disappear, we shall have

$$r_{,,}' + r_{,}'' - r_{,,}'' - r_{,}' = 2\,l,$$

and the expression of the force will become

$$\frac{i i' l}{a},$$

which is reduced to $i i'$ when the length l is equal to the distance a.

[To be continued.]

LXI. *Proceedings of Learned Societies.*

ROYAL SOCIETY.

THE meetings of this Society for the session 1825–6 were resumed on the 17th instant; when the following papers were read:—On the changes that have taken place in some ancient alloys of copper, by John Davy, M.D. F.R.S.—Observations on the apparent positions and distances of 468 double and triple fixed stars, made at the observatory at Passy,

THE PHILOSOPHICAL MAGAZINE AND JOURNAL:

COMPREHENDING

THE VARIOUS BRANCHES OF SCIENCE, THE LIBERAL AND FINE ARTS, AGRICULTURE, MANUFACTURES, AND COMMERCE.

By RICHARD TAYLOR, F.L.S.

MEMBER OF THE ASTRONOMICAL SOCIETY OF LONDON, OF THE METEOROLOGICAL SOCIETY; AND OF THE ROYAL ASIATIC SOCIETY OF GREAT BRITAIN AND IRELAND.

"Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes." JUST. LIPS. *Monit. Polit.* lib. i. cap. 1.

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To enable us to judge of the validity of this supposition, put $y' = y + (y' - y)$ in the formula (3); then

$$y = \frac{y}{4\pi} \cdot \int \frac{(r^2 - a^2)\, a\, d\theta' \sin\theta'\, d\varpi'}{f^3} + \frac{1}{4\pi} \cdot \int \frac{(r^2 - a^2)\, a\, (y' - y)\, d\theta' \sin\theta'\, d\varpi'}{f^3}.$$

Now, the first term in the value of y is what results from M. Poisson's supposition, that the thickness of the molecule remains constant. That supposition therefore virtually admits the equality of the second term to zero. It is very plain that, if the second term be not equal to zero, we shall not obtain the exact value of the double fluent by integrating on the supposition that the thickness of the molecule is constant. Now it is to prove that the second term in the foregoing value of y is evanescent, that Laplace has taken so much pains without having given a satisfactory demonstration of it. It has likewise been shown above, that the evanescence of the same quantity is in reality the foundation of the whole analytical theory. It would be superfluous to add another word respecting M. Poisson's demonstration, which affords no additional evidence of the proposition to be proved.

An attentive reader who considers the foregoing observations must allow that some material inadvertencies and inaccuracies have originally slipt into the analysis of Laplace. But the theory having been published, it has been deemed advisable to repel all objections, and to defend it *to the utterance.*

Jan. 6, 1826. JAMES IVORY.

[To be continued.]

IV. *Sequel of the Memoir of M.* AMPERE *on a new Electro-dynamic Experiment, on its Application to the Formula representing the mutual Action of the two Elements of Voltaic Conductors, and on new Results deduced from that Formula.*

[Concluded from vol. lxvi. p. 387.]

WE have found, in the applications which we have just made of the formula which expresses the mutual action of two infinitely small portions of voltaic conductors, (see page 385 of this memoir in the preceding volume of the Philosophical Magazine)

$$2\,\frac{dM}{d\theta}\,d\theta = -\,a i i' (\cos\theta - \sin\theta)\left(\frac{1}{\sin^2\theta\cos^2\theta} + \frac{1}{\sin\theta\cos\theta} + 1\right) d\theta$$

for the differential momentum of rotation in virtue of which a rectilinear conductor, of which the length is $2a$, moveable around

around its centre, oscillates from side to side of its situation of equilibrium, when it is submitted to the action of two fixed conductors, each of which has one of its extremities at this centre, and whose length is a. In the instrument which I have contrived for verifying this result of my formula, it is not only these two conductors which act on that which is moveable, but also the circular portion of the voltaic circuit which joins the two other extremities of the fixed conductors: as the action which results from this portion is exerted in a contrary direction, a momentum is obtained of which the sign is opposed to that of the momentum of which we have just obtained the value, it must be added to the first; and what is very remarkable, the total momentum takes a form much more simple. In short, in naming M′ the momentum of rotation produced by this arc, that which must be added to $2\frac{d\,M}{d\,\theta}\,d\,\theta$

is evidently $$2\frac{d\,M'}{d\,\theta}\,d\,\theta;$$

as the radius of the arc s' is equal to a, we have $s' = 2\,a\,\theta + C$,

whence $$d\,\theta = \frac{d\,s'}{2\,a},$$

and, consequently, $$2\frac{d\,M'}{d\,\theta}\,d\,\theta = 4\,a\frac{d\,M'}{d\,s'}\,d\,\theta.$$

But the tangential force in the direction of the element $d\,s'$

being $\frac{1}{2}\,i\,i'\,d\,s'\,d\,\frac{\cos^2\beta}{r}$,

and its momentum of causing this element to turn round its centre being equal, and of a sign contrary to that whose value we are seeking, we have

$$\frac{d^2\,M'}{d\,s\,d\,s'}\,d\,s\,d\,s' = -\tfrac{1}{2}\,a\,i\,i'\,d\,s'\,d\,\frac{\cos^2\beta}{r},$$

whence $$\frac{d\,M'}{d\,s'}\,d\,s' = -\tfrac{1}{2}\,a\,i\,i'\left(\frac{\cos^2\beta''}{r''} - \frac{\cos^2\beta'}{r'}\right)d\,s'.$$

Observing that it is necessary to integrate in the same manner in relation to the direction of the current as for rectilinear fixed conductors, we find

$$\cos\beta' = -\cos\theta,\ r' = 2\,a\sin\theta,\ \cos\beta'' = \sin\theta,\ r'' = 2\,a\cos\theta;$$

thus

$$\frac{d\,M'}{d\,s'} = \tfrac{1}{4}\,i\,i'\left(\frac{\cos^2\theta}{\sin\theta} - \frac{\sin^2\theta}{\cos\theta}\right) = \tfrac{1}{4}\,i\,i'(\cos\theta - \sin\theta)\left(\frac{1}{\sin\theta\cos\theta} + 1\right),$$

and $$4\,a\frac{d\,M'}{d\,s'}\,d\,\theta = a\,i\,i'\,(\cos\theta - \sin\theta)\left(\frac{1}{\sin\theta\cos\theta} + 1\right)d\,\theta.$$

Uniting this momentum with that which we have called

$$2\frac{d\,M}{d\,s}\,d\,\theta,$$

we

we have $-\frac{a\,i\,i'\,(\cos\theta - \sin\theta)}{\sin^2\theta\,\cos^2\theta}\,d\,\theta = -\frac{4\,a\,i\,i'\sqrt{2}\,\sin\frac{1}{2}\,\eta}{\cos^2\eta}\,d\,\theta,$

because, besides the equation $\sin\theta\,\cos\theta = \frac{1}{2}\cos\frac{1}{2}\,\eta$ which we have deduced (page 394 of the former portion of this memoir) from the value of θ, $\theta = \frac{1}{2}\,\epsilon = \frac{1}{2}\left(\frac{\pi}{2} - \eta\right)$,

we obtain also from this same value

$$\cos\theta - \sin\theta = \sqrt{2}\,\sin\tfrac{1}{4}\,\eta.$$

The action which causes the moveable conductor to oscillate is then proportionate to the sine of the quarter of the angle comprised between the directions of the two fixed rectilinear conductors, divided by the square of the cosine of the half of the same angle; it becomes null with this angle, as it ought to be, and infinite when they are directed following the same right line, because then $\eta = \frac{\pi}{2}$.

In the instrument intended for the measurement of these oscillations, the two extremities of the moveable conductor are also joined by a conductor forming a semi-circumference; but account is only to be taken of the action exercised on its rectilinear portion; since the circuit formed by the two fixedre ctilinear conductors, and by the arc which joins the extremities of it, is a closed circle which cannot act on the circular portion of the moveable conductor.

The value which we have found for the elementary momentum

$$\frac{d\,M'}{d\,s'}\,d\,s' = -\tfrac{1}{2}\,a\,i\,i'\left\{\frac{\cos^2\beta''}{r} - \frac{\cos^2\beta'}{r}\right\}d\,s'$$

expresses generally the action impressed by the little arc $d\,s'$ on a conductor of any form whatever, so as to make it turn round an axis elevated by the centre of this arc perpendicularly to its plane: this action is then independent of the form of this conductor, and only depends on the situation of its two extremities relatively to the little arc $d\,s'$; it is equal, as it ought to be, to the produce of the radius a by the value which we have obtained (see vol. lxvi. p. 378) for the force which is exercised on the same moveable conductor by a small portion equal to $d\,s'$ of a rectilinear conductor directed according to this arc $d\,s'$. When we wish to see the action of an arc terminated, we must integrate afresh with relation to s', and this second integration generally gives a different result in the two cases; but this result is the same when the moveable conductor has one of its extremities in the axis, and the other on the circumference of which the arc s' makes a part. The only sign of the value which is obtained becomes changed, because

because in one case β augments with s', and diminishes in the other; for then the angle β' is a right angle, and the angle β'' is comprised between a chord and a tangent formed by the extremity, whence it is easy to conclude

$$r = 2\,a \sin\beta,\; s' = c - 2\,a\,\beta,\; \mathrm{d}\,s' = -2\,a\,\mathrm{d}\,\beta,$$

which gives $$\frac{\mathrm{d}\,s'}{r} = -\frac{\mathrm{d}\,\beta}{\sin\beta},$$

and for the value of the momentum sought

$$\tfrac{1}{2}\,a\,i\,i' \int \frac{\cos^2\beta\,\mathrm{d}\,\beta}{\sin\beta},$$

which is precisely the same form as that of the force in the case of the rectilinear conductor, and is integrated precisely in the same manner. The reason of this analogy between these two cases, otherwise so different, is found in this circumstance,—that in that of the rectilinear conductor we had

$$r = \frac{a}{\sin\beta},\; s = -a\cot\beta,\; \mathrm{d}\,s' = \frac{a\,\mathrm{d}\,\beta}{\sin^2\beta};$$

whence we obtain $$\frac{\mathrm{d}\,s'}{r} = \frac{\mathrm{d}\,\beta}{\sin\beta},$$

which differs only by the signs of the value of $\frac{\mathrm{d}\,s'}{r}$ in the case of the circular conductor; which ought to be so, because in the first, β diminishes when s' augments, and because it augments with s' in the second.

Let us now consider two rectilinear conductors the directions of which form a right angle, but may not be situated in the same plane, by naming a the right line which measures the distance of these directions, and by taking the points where they are met by the right line a for the origin of s and of s', we have $$r^2 = a^2 + s' + s'^2,\; r\,\frac{\mathrm{d}\,r}{\mathrm{d}\,s'}\,\mathrm{d}\,s' = s'\,\mathrm{d}\,s',$$

and $$\cos\beta = -\frac{\mathrm{d}\,r}{\mathrm{d}\,s'} = -\frac{s'}{r}.$$

But we have seen (vol. lxvi. page 381) that the mutual action of the two elements d s and d s' is generally equal to

$$\tfrac{1}{2}\,i\,i'\,\frac{\mathrm{d}\,s'}{\cos\beta}\,\mathrm{d}\,\frac{\cos^2\beta}{r};$$

it may then be written thus,

$$\tfrac{1}{2}\,i\,i'\,r\,s'\,\mathrm{d}\,s'\,\mathrm{d}\,\frac{1}{r^3};$$

and as this force must be multiplied by $\frac{a}{r}$ to have its component parallel to the right line a, the value of this component is found to be $$-\tfrac{1}{2}\,a\,i\,i'\,s'\,\mathrm{d}\,s'\,\mathrm{d}\,\frac{1}{r^3},$$

by

by integrating it with relation to s between two points whose distances to the element $\mathrm{d}\, s'$ may be r' and r'', we have.

$$-\tfrac{1}{2}\, a\, i\, i'\, s'\, \mathrm{d}\, s' \left(\frac{1}{r''^3} - \frac{1}{r'^3}\right),$$

which may be written thus

$$-\tfrac{1}{2}\, a\, i\, i' \left(\frac{1}{r''^2} \cdot \frac{\mathrm{d}\, r''}{\mathrm{d}\, s'}\, \mathrm{d}\, s' - \frac{1}{r'^2} \cdot \frac{\mathrm{d}\, r'}{\mathrm{d}\, s'}\, \mathrm{d}\, s'\right),$$

of which the integral, taken for the first term of $r_{\prime}''$ to $r_{\prime\prime}''$, and for the second of $r_{\prime}'$ to $r_{\prime\prime}'$, gives

$$\tfrac{1}{2}\, i\, i' \left(\frac{a}{r_{\prime\prime}''} - \frac{a}{r_{\prime}''} - \frac{a}{r_{\prime\prime}'} + \frac{a}{r_{\prime}'}\right),$$

so that the action sought is precisely the same as if it were produced by four forces equal to $\tfrac{1}{2}\, i\, i'$, directed according to the right lines which join two by two the extremities of the conductors, two of these forces being attractive and the other two repulsive.

If there be required the momentum of rotation impressed in the case which we are here examining, by one of the two rectilinear conductors on the other conductor around an axis parallel to the first, and whose shortest distance to the line which we have named a be represented by b, it will be necessary to multiply the component parallel to a of the mutual action of the two elements by $s' - b$, and then integrate in the same manner: as s' is constant in the first integration, it will suffice to perform this multiplication after it has been executed; thus we shall have two terms of the same form to integrate anew, the first will be

$$-\tfrac{1}{2}\, a\, i\, i'\, \frac{s'-b}{r''^2} \cdot \frac{\mathrm{d}\, r''}{\mathrm{d}\, s'}\, \mathrm{d}\, s',$$

and there will come, by integrating partially,

$$\tfrac{1}{2}\, a\, i\, i'\, \frac{s'-b}{r''} - \tfrac{1}{2}\, a\, i\, i' \int \frac{\mathrm{d}\, s'}{r''}.$$

But it is easy to see that by naming c the value of s which corresponds to r'', and which is a constant in the actual integration, we have

$$r'' = \frac{\sqrt{a^2+c^2}}{\sin \beta''},\ s' = -\sqrt{a^2 + c^2}\cot \beta'',\ \mathrm{d}\, s' = \frac{\sqrt{a^2+c^2}}{\sin^2 \beta''}\, \mathrm{d}\, \beta'',$$

thus $$\int \frac{\mathrm{d}\, s'}{r''} = \int \frac{\mathrm{d}\, \beta''}{\sin \beta''} = \mathrm{l}\, \frac{\operatorname{tang} \tfrac{1}{2} \beta_{\prime\prime}''}{\operatorname{tang} \tfrac{1}{2} \beta_{\prime}''};$$

the second term will be integrated in the same manner, and we shall have at last, for the momentum of rotation sought,

$$\tfrac{1}{2}\, a\, i\, i' \left(\frac{s_{\prime\prime}'-b}{r_{\prime\prime}''} - \frac{s_{\prime}'-b}{r_{\prime}''} - \frac{s_{\prime\prime}'-b}{r_{\prime\prime}'} + \frac{s_{\prime}'-b}{r_{\prime}'} - \mathrm{l}\, \frac{\operatorname{tang} \tfrac{1}{2} \beta_{\prime\prime}'' \operatorname{tang} \tfrac{1}{2} \beta_{\prime}'}{\operatorname{tang} \tfrac{1}{2} \beta_{\prime}'' \operatorname{tang} \tfrac{1}{2} \beta_{\prime\prime}'}\right).$$

In the case where the axis of rotation parallel to the right line s passes by the point of intersection of the two right lines a and

a and s', we have $b = 0$; and if we suppose, besides, that the current which flows along s' departs from this point of intersection, we shall moreover have

$$s_/' = 0,\ \beta_/' = \frac{\pi}{2},\ \beta_/'' = \frac{\pi}{2},$$

so that the value of the momentum of rotation will be reduced to

$$\tfrac{1}{2}\, a\, i\, i' \left(\frac{s_{//}''}{r_{//}''} - \frac{s_{//}'}{r_{//}'} - \mathrm{l}\, \frac{\mathrm{tang}\ \frac{1}{2}\beta_{//}''}{\mathrm{tang}\ \frac{1}{2}\beta_{//}'} \right).$$

We have just seen that when the directions of the two rectilinear conductors of which we seek the mutual action, form a right angle, that of the two elements of s and s' becomes reduced to

$$-\tfrac{1}{2}\, i\, i'\, r\, s'\, \mathrm{d}s'\, \mathrm{d}\frac{1}{r^3},$$

and that we have, in the same case,

$$r = \sqrt{a^2 + s^2 + s'^2}:$$

then this elementary action may be thus written,

$$-\tfrac{1}{2}\, i\, i'\, s'\, \mathrm{d}s' \sqrt{a^2 + s^2 + s'^2}\ \mathrm{d}\,(a^2 + s^2 + s'^2)^{-\frac{3}{2}}$$
$$= \tfrac{3}{2}\, i\, i'\, \frac{s\, s'\, \mathrm{d}s\, \mathrm{d}s'}{(a^2 + s^2 + s'^2)^2}.$$

As it acts in the direction of the right line r, it is necessary, to find the momentum of rotation which results from it around the right line a, to multiply it by the sine of the angle contained between its direction and that of this right line, which is equal to

$$\frac{\sqrt{s^2 + s'^2}}{\sqrt{a^2 + s^2 + s'^2}},$$

and by the shortest distance

$$\frac{s\, s'}{\sqrt{s^2 + s'^2}},$$

that is to say, that the force must be multiplied by the quantity

$$\frac{s\, s'}{\sqrt{a^2 + s^2 + s'^2}},$$

which I shall represent by q, which gives

$$\frac{\mathrm{d}^2 M}{\mathrm{d}s\, \mathrm{d}s'}\, \mathrm{d}s\, \mathrm{d}s' = \tfrac{3}{2}\, i\, i'\, \frac{s^2\, s'^2\, \mathrm{d}s\, \mathrm{d}s'}{(a^2 + s^2 + s'^2)^{\frac{5}{2}}}.$$

This value at first does not appear easy to integrate; but if we distinguish the value of q once with relation to s, and the other by varying s', we have

$$\frac{\mathrm{d}q}{\mathrm{d}s} = \frac{s'}{\sqrt{a^2 + s^2 + s'^2}} - \frac{s'\, s^2}{(a^2 + s^2 + s'^2)^{\frac{3}{2}}} = \frac{a^2 s' + s'^3}{(a^2 + s^2 + s'^2)^{\frac{3}{2}}},$$

$$\frac{\mathrm{d}^2 q}{\mathrm{d}s\, \mathrm{d}s'} = \frac{a^2 + 3 s'^2}{(a^2 + s^2 + s'^2)^{\frac{3}{2}}} - \frac{3(a^2 + s'^2)\, s'^2}{(a^2 + s^2 + s'^2)^{\frac{5}{2}}} =$$

$$\frac{a^4}{(a^2 + s^2 + s'^2)^{\frac{3}{2}}} + \frac{3 s^2 s'^2}{(a^2 + s^2 + s'^2)^{\frac{5}{2}}},$$

so that $\frac{d^2 M}{ds\,ds'}\,ds\,ds' = \frac{1}{2}\,ii'\left[\frac{d^2 q}{ds\,ds'}\,ds\,ds' - \frac{a^2\,ds\,ds'}{(a^2+s^2+s'^2)^{\frac{3}{2}}}\right]$;

the quantity $$\frac{a^2\,ds\,ds'}{(a^2+s^2+s'^2)^{\frac{3}{2}}}$$

integrated first with relation to s, so that the integral becomes null with s, gives

$$\frac{a^2\,s\,ds'}{(a^2+s'^2)\sqrt{a^2+s^2+s'^2}},$$

that it remains to integrate by only varying s', the most simple means to come there is to make

$$\sqrt{a^2+s^2+s'^2} = \sqrt{u} - s',$$

which gives

$$s = \frac{u-a^2-s^2}{2\sqrt{u}},\quad \frac{ds'}{\sqrt{a^2+s^2+s'^2}} = \frac{du}{2u},$$

$$a^2 + s'^2 = \frac{(u-a^2-s^2)^2+4a^2u}{4u} = \frac{(u+a^2-s^2)^2+4a^2s^2}{4u},$$

and changes the quantity to integrate into

$$\frac{\frac{a\,du}{2as}}{1+\frac{(u+a^2-s^2)^2}{4a^2s^2}},$$

of which the integral, taken so that it vanishes when $s=0$ is

$$a\left\{\text{arc tang}\,\frac{(s'+\sqrt{a^2+s^2+s'^2})^2+a^2-s^2}{2as} - \text{arc tang}\,\frac{a}{s}\right\},$$

which becomes reduced, by executing the indicated operations and in calculating by the formula known the tangent of the difference of the two arcs, to

$$a\ \text{arc tang}\,\frac{ss'}{a\sqrt{a^2+s^2+s'^2}} = a\ \text{arc tang}\,\frac{q}{a}.$$

We have then for the value of the momentum M of rotation, in the case where the two electric currents, of which the lengths are s and s', depart from points where their directions meet the right line which measures the shortest distance from it,

$$M = \frac{1}{2}\,ii'\left(q - a\ \text{arc tang}\,\frac{q}{a}\right),$$

when $a=0$, we have evidently $M = \frac{1}{2}\,i\,i'\,q$, that which agrees with the value $M = \frac{1}{2}\,i\,i'p$ which we have already found (page 382), because then q becomes the perpendicular which was then distinguished by p. If we suppose a infinite, M becomes null, as it should be, because that in this case a arc tang $\frac{q}{a} = q$.

If we name z the angle of which the tangent is

$$\frac{ss'}{a\sqrt{a^2+s^2+s'^2}},$$

we

we shall have $$M = \frac{1}{2} i i' q \left(1 - \frac{z}{\text{tang}\, z}\right);$$

it is the value of the momentum of rotation which would be produced by a force equal to

$$\frac{1}{2} i i' \left(1 - \frac{z}{\text{tang}\, z}\right),$$

acting according to the right line which joins the two extremities of the conductors opposed to those where they are met by the right line which measures the shortest distance of it.

It is, for the rest, easy to see that if, instead of supposing that the two currents depart from the point where they meet the right, we had made the calculations for what limits soever, we should have found a value of M composed of four terms of the form of that which we have obtained in this particular case, two of these terms being positive and two negative.

By combining the last result which we have just obtained with that which we found immediately before, it is easy to calculate the momentum of rotation resulting from the action of a conductor having for its form the perimeter of a rectangle, and acting on a moveable conductor around one of the sides of a rectangle, when the direction of this conductor is perpendicular to the plane of the rectangle, whatever in other respects be its distance from the other sides of the rectangle, and the dimensions of this one. In determining by experiment the instant when the moveable conductor is in equilibrium between the opposed actions of the two rectangles situated in the same plane, but of different sizes and at different distances of the moveable conductor, we have a very simple means of procuring verifications of my formula susceptible of great precision: it is that which we may easily make with the instrument of which I spoke above, by conveniently modifying the fixed conductors which make a part of it.

The same calculations may be made for any value whatsoever of the angle of the directions of the two rectilinear conductors: by naming this angle ε, we have

$$r = \sqrt{a^2 + s^2 + s'^2 - 2 s s' \cos \varepsilon},$$

and in always representing by q the quantity $\frac{s s'}{r}$, we find that the force parallel to the right line a is equal to

$$\frac{1}{2} i i' \left(\frac{a}{r} + a \cos \varepsilon \iint \frac{d s\, d s'}{r^3}\right).$$

The momentum of rotation around the right line a is then equal to

$$\frac{1}{2} i i' \left(q \sin \varepsilon - r \cot \varepsilon - \frac{a^2}{\sin \varepsilon} \iint \frac{d s\, d s'}{r^3}\right).$$

As

As to the integral which enters into these expressions

$$\iint \frac{ds \cdot ds'}{r^3} = \int \frac{(s - s' \cos \iota)\, ds'}{(a^2 + s'^2 \sin^2 \iota) \cdot \sqrt{a^2 + s^2 + s'^2 - 2ss' \cos \iota}},$$

we may obtain by the known method of integration of differentials which comprise a radix of the second order, and more easily by a particular process which I shall explain elsewhere.

V. *On the Theory of Evaporation.* *By* Thos. Tredgold, *Esq.* *To Mr. R. Taylor.*

Sir,

EVAPORATION has been considerably attended to, but rather as a matter of experimental research than with the object of finding those first principles which are essential to the process. In the following inquiry it is not intended to limit it to a particular case, but simply for illustration the vapour is supposed to be from the surface of water.

When the air in contact with water is saturated with vapour, evaporation ceases, or there is an equilibrium between the powers which produce and retard the formation of vapour.

Now conceive a portion of the vapour to be abstracted from the air, then the equilibrium will be destroyed; and all other circumstances being the same, the tendency to restore the equilibrium must be proportional to the quantity of vapour removed from the previously saturated air; for no other circumstance than the weight of vapour in a given portion of air is altered.

But, the equilibrium being destroyed, evaporation commences, and the vapour cannot be formed without a constant supply of heat; therefore, to obtain this supply of heat when there is no other source than the surrounding bodies of the same temperature, the temperature of the surface where the vapour forms must be depressed, in order that heat may flow to it from the adjoining bodies, or parts of the same body; and as the heat required is proportional to the quantity of vapour formed in a given time, the depression of temperature will be proportional to that quantity.

It will also be obvious that the vapour formed will be of the elasticity corresponding to the temperature of the surface producing it, and therefore will correspond to the depressed temperature of the evaporating surface.

Let T be the general temperature, t the temperature of the evaporating surface at its ultimate depression, and w the weight of vapour in grains that would saturate a cubic foot of air at the

Fig. 1.

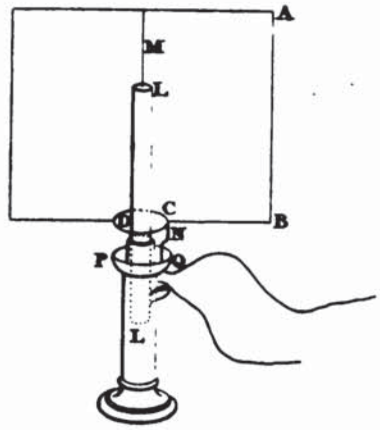


Fig. 2.

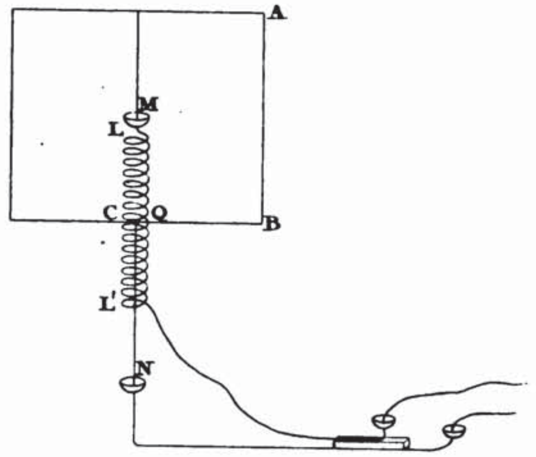


Fig. 4.

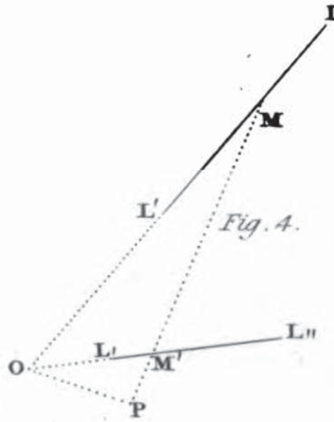


Fig. 3.

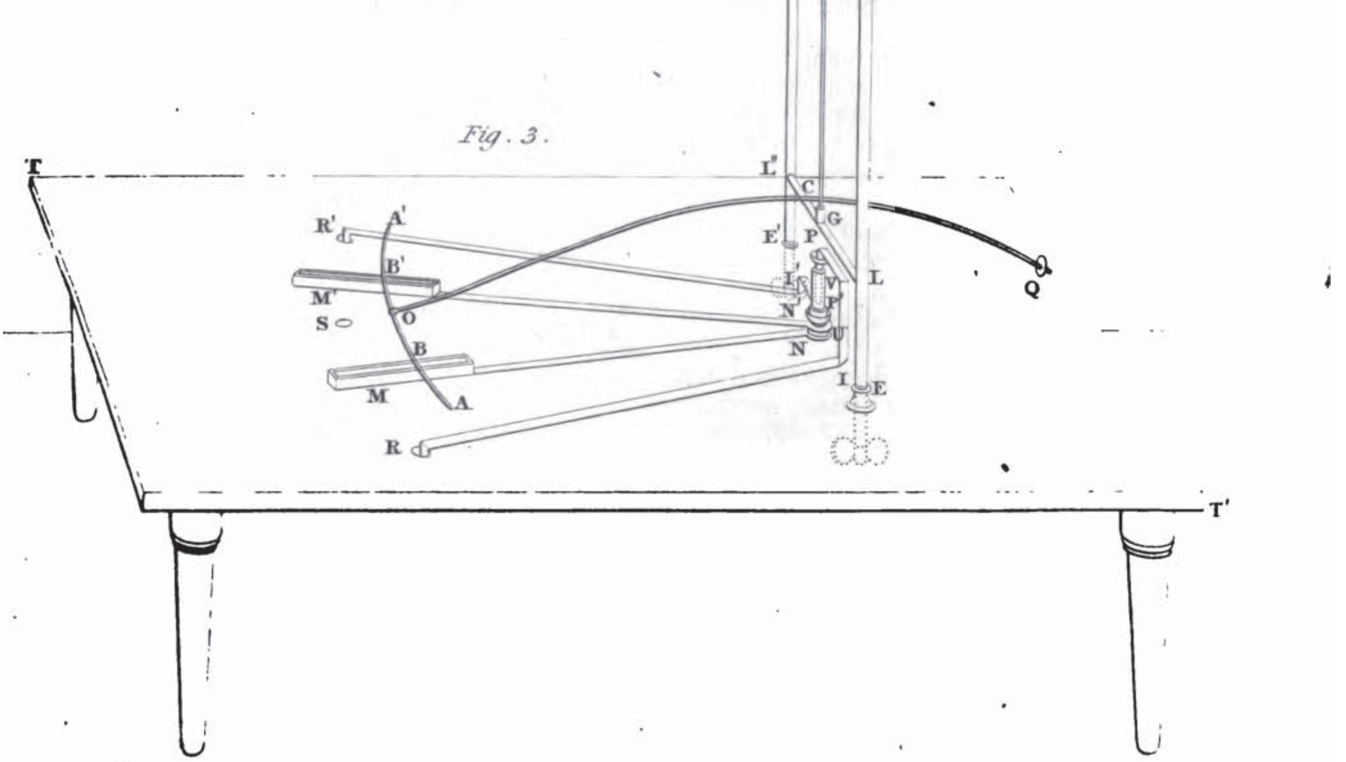


Fig. 5.

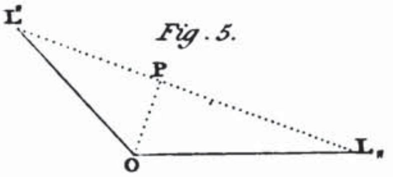
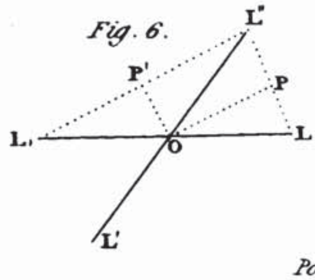


Fig. 6.



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